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Study of Solution as Locally Asymptotic Attractivity for Nonlinear Functional Integral Equation

Shantaram N Salunkhe

***Abstract:** In this paper we proved an existence result for local asymptotic attractivity of the solution for a nonlinear functional integral equation under certain condition which gives the existence as well as existence of the asymptotic stability of solutions. An example is providing for indicating the natural realizations of abstract theory presented in the paper.*

***Keywords:** Solution of Functional Integral Equation, Fixed Point Theorem, Locally Asymptotic Attractive Solution.*
***Subject Classifications:** 47H10, 34A60*

I. INTRODUCTION

Let $\mathbb{R}$ be the real line and I be the set of non-negative real numbers. Consider the nonlinear functional integral equations (In short FIE) of mixed type

$$x(t) = \left[f\left(t, x\big(\theta(t)\big)\right)\right]\left[\gamma(t) + \int_0^{\lambda(t)} g\left(t, x\big(\phi(s)\big)\right) ds\right] \quad \dots \quad (1)$$

for all $t, s \in I$, where $\gamma: I \to \mathbb{R}$, $f: I \times \mathbb{R} \to \mathbb{R}$, $g: I \times I \to \mathbb{R}$ and $\theta, \phi, \lambda: I \to I$.

Let $C(I, \mathbb{R})$ be the space of continuous real-valued functions on I. By a solution of the FIE (1.1) we mean a function $x \in C(I, \mathbb{R})$ that satisfies the equation FIE (1.1).

The FIE (1.1) is discussed in the literature for different aspects of the solutions. The mentioned equation has general form and contains as particular cases a lot of functional equations and nonlinear integral equations of Volterra type.

The details of these considerations appear in the monographs like Krasnoselskii [1], Väth [2], Burton [3] and Dhage [4] etc. The main tools used in the considerations are the techniques of fixed point theorem of Schauder [5], Krasnoselskii [1], and Dhage [4], etc. The global asymptotic stability of the nonlinear functional integral equation in the different form and [6]

$$x(t) = f\left(t, x\big(\theta(t)\big)\right) + \int_0^{\lambda(t)} g\left(t, x\big(\phi(s)\big)\right) ds \quad \dots \quad (2)$$

has been discussed in Banas and Dhage [7] via a measure of noncompactness defined in Banas [9] etc. The characterization of the measure of noncompactness itself gives the uniform local asymptotic attractivity via a hybrid fixed point theorem of Dhage [6] which gives the asymptotic stability of the solution for FIE (1.1) [8].

II. AUXILIARY RESULTS

Let $X = BC(I, \mathbb{R})$ be the space of continuous and bounded real-valued functions on I and $\Omega \subseteq X$. Let $Q: X \to X$ be an operator and consider the following operator equation in X

$$x(t) = Qx(t) \quad \dots \quad (3)$$

for all $t \in I$. Below given different characterizations of the solutions for the operator equations $x(t) = Qx(t)$, $t \in I$. [10]

Definition (2.2): We say that solutions of $x(t) = Qx(t)$ are locally attractive if $\exists\, x_0 \in BC(I, \mathbb{R})$ and $r > 0$ such that for all solutions $x = x(t)$ and $y = y(t)$ of equation $x(t) = Qx(t)$ belonging to closed ball $\overline{\mathcal{B}_r}(x_0) \cap \Omega$ we have that

$$\lim_{t\to\infty}[x(t) - y(t)] = 0 \quad \dots \quad (4)$$

In the case, when for each $\in > 0$, $\exists\, T > 0$such that

$$|x(t) - y(t)| \leq \in \quad \dots \quad (5)$$

for all $x, y \in \overline{\mathcal{B}_r}(x_0) \cap \Omega$ being solution of $x(t) = Qx(t)$ for $t \geq T$. We say that solution of equation $x(t) = Qx(t)$ are uniformly locally attractive on I.

In this paper, we will study with only the local characterization of solutions for the equation (2.1) on I.

Definition (2.5): A solution $x \in BC(I, \mathbb{R})$ to the equation $x(t) = Qx(t)$ is said to be asymptotic if $\exists\, c \in I$ such that $\lim_{t\to\infty} x(t) = c$, then we say that the solution x is asymptotic to c on I.

Now the following definition useful in the sequel.

Definition (2.6): The solution of equation $x(t) = Qx(t)$ are said to be locally asymptotically attractive if $\exists\, x_0 \in BC(I, \mathbb{R})$ and $r > 0$ such that for all asymptotic solutions $x = x(t)$,

$y = y(t)$ of equation belonging to $\overline{\mathcal{B}_r}(x_0) \cap \Omega$. In the case when if for every $\epsilon > 0, \exists\, T > 0$ such that $|x(t) - y(t)| \leq \epsilon$ is satisfied for all $x, y \in \overline{\mathcal{B}_r}(x_0) \cap \Omega$ being asymptotic solutions of equation $x(t) = Qx(t)$ and for $t \geq T$ we say that solutions of the equation $x(t) = Qx(t)$ are uniformly locally asymptotic attractive.

Remark: Every locally asymptotically attractive solution is asymptotically attractive, but the

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*Correspondence Author(s)
Dr. Shantaram N Salunkhe*, Associate Professor and Head, Department of Mathematics, Rani Laxmibai Mahavidyalaya Parola Dist. Jalgaon, (Maharashtra), India. Email ID: saisns@rediffmail.com, ORCID ID: 0009-0009-6527-6626

converse may not be true [11].

We seek the solution of the FIE (1.1) in the space $BC(I, \mathbb{R})$ of continuous and bounded real valued functions defined on I [12]. Define a standard supremum norm $\|\cdot\|$ and a multiplication '$\cdot$' in $BC(I, \mathbb{R})$ by

$\|x\| = \sup_{t\in I}|x(t)|$ and $(x \cdot y)(t) = x(t) \cdot y(t),\ t \in I$

Clearly $BC(I, \mathbb{R})$ becomes a Banach algebra with norm $\|\cdot\|$ and '$\cdot$' in it. By $L'(I, \mathbb{R})$ denote the space of Lebesgue integrable functions on I and the norm $\|\cdot\|_{L'}$ in $L'(I, \mathbb{R})$ is defined by

$$\|x\|_{L'} = \int_0^\infty |x(t)|ds$$

Definition (2.7): An operator $Q: X \to X$ is called Lipschitz if $\exists$ a constant $k > 0$ such that

$$\|Qx - Qy\| \le k\|x - y\|$$

For all $x, y \in X$ and constant k is called Lipschitz constant of Q on X.

Definition (2.8): An operator Q on a Banach space X into itself is called compact if for any bounded subset S of $X, Q(S)$ is a relatively compact subset of X. If Q is continuous and compact then it is called completely continuous on X.

We use the following hybrid fixed point theorem of Dhage [7] for proving the existence result for uniform local asymptotic attractivity of the solution for the FIE (1.1).

Theorem 2.9 (Dhage [6]): Let S be a closed convex and bounded subset of the Banach algebra X and let $A, B: S \to X$ be two operators such

a) A is Lipschitz with the Lipschitz constant k.
b) B is completely continuous.
c) $AxBx \in S$ for all $x \in S$, and
d) $Mk < 1$, where $M = \|B(S)\| = \sup\{\|Bx\|: x \in S\}$.

Then operator equation $AxBx = x$ has a solution and the set of all solutions is compact in S.

III. HYPOTHESES AND MAIN RESULTS

We consider the following set of hypotheses in the sequel.

(H_1) The function $\theta: I \to \mathbb{R}$ is continuous.

(H_2) The function $f: I \times \mathbb{R} \to \mathbb{R}$ is continuous and $\exists$ a bounded function $\ell: I \to I$ with bound satisfying $|f(t,x) - f(t,y)| \le \ell(t)|x - y|$ for all $t \in I$, $x, y \in \mathbb{R}$.

(H_3) The function $F: I \to \mathbb{R}$ defined by $F(t) = |f(t, 0)|$ is bounded with $F_0 = \sup_{t\ge 0}F(t)$.

(H_4) The function $\phi: I \to I$ is measurable and the function $\lambda: I \to I$ is continuous.

(H_5) The function $\gamma: I \to I$ is continuous and $\lim_{t\to\infty}\gamma(t) = 0$.

(H_6) The function $g: I \times I \to \mathbb{R}$ is continuous and there exist continuous function $a, b: I \to I$

Satisfying $|g(t,s)| \le a(t)b(s)$ for all $t, s \in I$ where $\lim_{t\to\infty}a(t)\int_0^{\lambda(t)} b(s)ds = 0$.

Remark (3.1): Note that if the hypotheses $(H_4) - (H_6)$ hold, there exist constant

$M_1 > 0, M_2 > 0$ such that $M_1 = \sup_{t\ge 0}\gamma(t)$ and $M_2 = \sup_{t\ge 0}v(t) = \sup_{t\ge 0}\left[a(t)\int_0^{\lambda(t)} b(s)ds\right]$.

Theorem 3.1: Assume that the hypotheses (H_1) through (H_6) hold. Further if

$L(M_1 + M_2) < 1$, where the numbers M_1 and M_2 are defined in above remark (3.1) then the FIE (1.1) has at least one solution of FIE (1.1) are locally asymptotically attractive in I.

Proof: Let $X = BC(I, \mathbb{R})$. Let $\overline{\mathcal{B}}_r(0)$ be the closed ball in X centered at origin 0and radius r, where $r = \frac{F_0(M_1+M_2)}{1-L(M_1+M_2)} > 0$.

Define two mapping A and B on $\overline{\mathcal{B}}_r(0)$ by

$$Ax(t) = f\left(t, x\left(\theta(t)\right)\right) \quad \dots \quad (6)$$

$$\text{and} \quad Bx(t) = \gamma(t) + \int_0^{\lambda(t)} g\left(t, x\left(\phi(s)\right)\right) ds \quad \dots \quad (7)$$

for $t \in I$. By hypothesis (H_2), the mapping A is well defined and function Ax is continuous and bounded on I. Also due to functions γ and λ are continuous on I, the function Bx is also continuous and bounded by hypothesis (H_3). Hence A,B define the operators $A, B: \overline{\mathcal{B}}_r(0) \to X$. We shall show that A and B satisfy the conditions in theorem (2.9) on $\overline{\mathcal{B}}_r(0)$.

Here, we show that A is Lipschitz on $\overline{\mathcal{B}}_r(0)$. Let $x, y \in \overline{\mathcal{B}}_r(0)$. Then by hypothesis (H_2)

$$|Ax(t) - Ay(t)| = \left|f\left(t, x\left(\theta(t)\right)\right) - f\left(t, y\left(\theta(t)\right)\right)\right|$$
$$= \ell(t)\left|x\left(\theta(t)\right) - y\left(\theta(t)\right)\right|$$
$$= L\|x - y\|$$

for all $t \in I$.

Taking maximum t, we get

$$\|Ax - Ay\| \le L\|x - y\| \quad \text{for all } x, y \in \overline{\mathcal{B}}_r(0)$$

This shows that A is Lipschitz on $\overline{\mathcal{B}}_r(0)$ with the Lipschitz constant L.

Now, we show that B is continuous on $\overline{\mathcal{B}}_r(0)$.

Let $\epsilon > 0$ and $x, y \in \overline{\mathcal{B}}_r(0)$ such that $\|x - y\| < \epsilon$. Then we get

$$|Bx(t) - By(t)| = \left|\int_0^{\lambda(t)} g\left(t, x\left(\phi(s)\right)\right) ds - \int_0^{\lambda(t)} g\left(t, y\left(\phi(s)\right)\right) ds\right|$$
$$\le \int_0^{\lambda(t)} \left|g\left(t, x\left(\phi(s)\right)\right) - g\left(t, y\left(\phi(s)\right)\right)\right| ds$$
$$\le \int_0^{\lambda(t)} \left[\left|g\left(t, x\left(\phi(s)\right)\right)\right| + \left|g\left(t, y\left(\phi(s)\right)\right)\right|\right] ds$$
$$\le 2\int_0^{\lambda(t)} a(t)b(s)\, ds \qquad \text{as } (H_6)$$
$$\le 2\vartheta(t) \quad \dots \quad (8)$$

hence by hypothesis (H_6), $\exists\ T > 0$ such that $\vartheta(t) \le \epsilon$ for $t \ge T$. Thus for $t \ge T$, from (3.3) we have

$$|Bx(t) - By(t)| \le 2\epsilon.$$

Furthermore, let us assume that $t \in [0, T]$. Then similarly as above we obtain

$$|Bx(t)-By(t)| \le \int_0^{\lambda(t)} \left|g\left(t,x(\phi(s))\right)-g\left(t,y(\phi(s))\right)\right| ds$$
$$\le \int_0^{\lambda(t)} \omega_r^T(q,\epsilon)ds$$
$$\le \lambda_T \omega_r^T(q,\epsilon) \quad \ldots \quad (9)$$

where we denoted $\lambda_T = \sup\{\lambda(t): t \in [0,T]\}$ and $\omega_r^T(q,\epsilon) = \sup\{|g(t,x(s))g(t,y(s))|: t,s \in [0,T],,x,y \in [-r,r], |x-y| \le \epsilon\}$

Obviously, we have in view of continuity of λ that $\lambda_T < \infty$. We derive that $\omega_r^T(q,\epsilon) \to 0$ as $\epsilon \to 0$ and the above mentioned facts we conclude that the operator B maps continuously the ball $\overline{\mathcal{B}}_r(0)$ into itself.

Next, to show that the operator B is compact on $\overline{\mathcal{B}}_r(0)$, it is enough to show that every sequence $\{Bx_n\}$ in $B(\overline{\mathcal{B}}_r(0))$ has a cauchy subsequence.

Now by (H_5) and (H_6)

$$|Bx_n(t)| \le \gamma(t) + \int_0^{\lambda(t)} \left|g\left(t,x_n(\eta(s))\right)\right| ds$$
$$\le M_1 + \vartheta(t) \le M_1 + M_2 \quad \ldots \quad (10)$$

for all $t \in I$. Taking maximum t, we obtain

$$\|Bx_n\| \le M_1 + M_2 \text{ for all } n \in N.$$

This shows that the sequence $\{Bx_n\}$ is uniformly bounded sequence in $B(\overline{\mathcal{B}}_r(0))$.

Now show that it is also equi continuous.

Let $\epsilon > 0$, $\lim_{t\to\infty} \gamma(t) = 0$, $\lim_{t\to\infty} \vartheta(t) = 0$, there are constants $T_1, T_2 > 0$ such that $|\gamma(t)| < \frac{\epsilon}{4}$ for all $t \ge T_2$. Let $T = \max\{T_1,T_2\}$. Let $t,\tau \in I$ be arbitrary. If $t,\tau \in [0,T]$ then we have

$$|Bx_n(t)-Bx_n(\tau)|$$
$$\le |\gamma(t)-\gamma(\tau)| + \left|\int_0^{\lambda(t)} g\left(t,x_n(\phi(s))\right)ds - \int_0^{\lambda(\tau)} g\left(\tau,x_n(\phi(s))\right)ds\right|$$
$$\le |\gamma(t)-\gamma(\tau)| + \left|\int_0^{\lambda(t)} g\left(t,x_n(\phi(s))\right)ds - \int_0^{\lambda(\tau)} g\left(\tau,x_n(\phi(s))\right)ds\right| + \left|\int_0^{\lambda(\tau)} g\left(\tau,x_n(\phi(s))\right)ds - \int_0^{\lambda(\tau)} g\left(\tau,x_n(\phi(s))\right)ds\right|.$$
$$\le |\gamma(t)-\gamma(\tau)| + \int_0^{\lambda(t)} \left|g\left(t,x_n(\phi(s))\right)-g\left(\tau,x_n(\phi(s))\right)\right| ds + \left|\int_{\lambda(\tau)}^{\lambda(t)} \left|g\left(\tau,x_n(\phi(s))\right)\right| ds\right|$$
$$\le |\gamma(t)-\gamma(\tau)| + \int_0^{\lambda_T} \left|g\left(t,x_n(\phi(s))\right)-g\left(\tau,x_n(\phi(s))\right)\right| ds + |\vartheta(t)-\vartheta(\tau)|.$$

By the uniform continuity of the functions γ,ϑ on $[0,T]$ and the function g in $[0,T]\times[-r,r]$, $s \in [0,\lambda_T]$ we obtain $|Bx_n(t)-Bx_n(\tau)| \to 0$ as $t \to \tau$.

Here $\{Bx_n\}$ is an equicontinuous sequence of function in X. By an application of Arzela-Ascoli theorem yields that $\{Bx_n\}$ has a uniformly convergent subsequence on compact subset $[0,T]$ of $\mathbb{R}$ and call it to be the sequence itself. Now show that $\{Bx_n\}$ is cauchy in X.

Now $|Bx_n(t)-Bx(t)| \to 0$ as $n \to \infty\ \forall\ t \in [0,T]$.

Then for given $\epsilon > 0, \exists\ n_0 \in N$ such that $\sup_{0\le p\le T} \int_0^{\lambda(p)} \left|g\left(t,x_m(\phi(s))\right)-g\left(t,x_n(\phi(s))\right)\right| ds < \frac{\epsilon}{2}$ for all $m,n \ge n_0.$

If $m,n \ge n_0$ then we have

$$\|Bx_m - Bx_n\|$$
$$= \sup_{0\le t<\infty}\left|\int_0^{\lambda(p)} \left|g\left(t,x_m(\phi(s))\right)-g\left(t,x_n(\phi(s))\right)\right| ds\right|$$
$$\le \sup_{0\le p\le T}\left|\int_0^{\lambda(p)} \left|g\left(t,x_m(\phi(s))\right)-g\left(t,x_n(\phi(s))\right)\right| ds\right|$$
$$+ \sup_{p\ge T}\int_0^{\lambda(p)} \left[g\left(t,x_m(\phi(s))\right)+g\left(t,x_n(\phi(s))\right)\right] ds$$
$$< \frac{\epsilon}{2}+\frac{\epsilon}{2} = \epsilon$$

This shows that $\{Bx_n\} \subset B(\overline{\mathcal{B}}_r(0)) \subset X$ is cauchy. Since X is complete, $\{Bx_n\}$ converges to a point in X. As $B(\overline{\mathcal{B}}_r(0))$ is closed $\{Bx_n\}$ converges to a point in $B(\overline{\mathcal{B}}_r(0))$.

Hence $B(\overline{\mathcal{B}}_r(0))$ is relatively compact and consequently B is a continuous and compact operator on $\overline{\mathcal{B}}_r(0)$.

Next, we show that $AxBx \in \overline{\mathcal{B}}_r(0)$ for all $x \in \overline{\mathcal{B}}_r(0)$. Let all $x \in \overline{\mathcal{B}}_r(0)$ be arbitrary. Then

$$|Ax(t)Ax(t)| \le |Ax(t)||Ax(t)|$$
$$\le \left|f\left(t,x(\phi(t))\right)\right|\left(|\gamma(t)| + \int_0^{\lambda(t)} \left|g\left(t,x(\phi(s))\right)\right| ds\right)$$
$$\le \left[\left|f\left(t,x(\phi(t))\right)-f(t,o)\right| + |f(t,o)|\right]\left(|\gamma(t)| + a(t)\int_0^{\lambda(t)} b(s)ds\right)$$
$$\le [\ell(t)|x(\phi(t))| + F(t)](|\gamma(t)| + \vartheta(t))$$
$$\le [L|x(\phi(t))| + F_0](M_1+M_2)$$
$$\le L(M_1+M_2)\|x\| + F_0(M_1+M_2)$$
$$= \frac{F_0(M_1+M_2)}{1-L(M_1+M_2)} = r$$

for all $t \in \mathbb{R}_+$. Taking the maxima over t, we obtain $\|AxBx\| \le r$ for all $x \in \overline{\mathcal{B}}_r(0)$.

Hence hypothesis (c) of theorem (2.9) holds. Therefore, here one has

$$M = \|B(\overline{\mathcal{B}}_r(0))\| = \sup\{\|Bx\| : x \in \overline{\mathcal{B}}_r(0)\}$$
$$= \sup\left\{\sup_{t\geq 0}\left\{|\gamma(t)| + \int_0^{\lambda(t)}\left|g\left(t, x(\phi(s))\right)\right| ds\right\} : x \in \overline{\mathcal{B}}_r(0)\right\}$$
$$\leq \sup_{t\geq 0}|\gamma(t)| + \sup_{t\geq 0}\vartheta(t)$$
$$\leq M_1 + M_2$$
and therefore, $MK = L(M_1 + M_2) < 1$.

Now apply theorem (2.9) to conclude that the FIE (1.1) has a solution on $\mathbb{R}_+$. Now we show the uniform locally asymptotic attractivity of the solution for FIE (1.1). Let x and y be any two solutions of the FIE (1.1) in $\overline{\mathcal{B}}_r(0)$ defined on $\mathbb{R}_+$. Then we have $|x(t) - y(t)| \leq \left|f\left(t, x(\phi(t))\right)\left(\gamma(t) + \int_0^{\lambda(t)} g\left(t, x(\phi(s))\right) ds\right)\right| +$
$$\left|f\left(t, y(\phi(t))\right)\left(\gamma(t) + \int_0^{\lambda(t)} g\left(t, y(\phi(s))\right) ds\right)\right|$$
$$\leq \left|f\left(t, x(\phi(t))\right)\right|\left(|\gamma(t)| + \int_0^{\lambda(t)}\left|g\left(t, x(\phi(s))\right)\right| ds\right) + \left|f\left(t, y(\phi(t))\right)\right|\left(|\gamma(t)| + \int_0^{\lambda(t)}\left|g\left(t, y(\phi(s))\right)\right| ds\right)$$
$$\leq 2(Lr + F_0)(|\gamma(t) + \vartheta(t)|)$$
for all $t \in \mathbb{R}_+$. Since $\lim_{t\to\infty}\gamma(t) = 0$ and $\lim_{t\to\infty}\vartheta(t) = 0$ for $\epsilon > 0$, there are real number $T_1 > 0$ and $T_2 > 0$ such that $|\gamma(t)| < \frac{\epsilon}{2(Lr+F_0)}$ for all $t \geq T_1$ and $\vartheta(t) < \frac{\epsilon}{2(Lr+F_0)}$ for all $t \geq T_2$.

If choose $T^* = \mathrm{Max}\{T_1, T_2\}$, then from the above inequality it follows that $|x(t) - y(t)| \leq \epsilon$ for all $t \geq T^*$. It is easy to pro that every solution of the FIE (1.1) asymptotic to zero on $\mathbb{R}_+$. consequently the FIE (1.1) has a solution and all the solutions are uniformly locally asymptotically attractive on $\mathbb{R}_+$.The proof is complete.

IV. CONCLUSION

We have concluded from this research article that the solution exist for local asymptotic attractivity of the solution for a nonlinear functional integral equation under certain condition which gives the existence as well as existence of the asymptotic stability of solutions which will be useful to new researchers in mathematics.

DECLARATION STATEMENT

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AUTHOR'S PROFILE

Dr. Shantaram N Salunkhe, Associate Professor and Head, Department of Mathematics, Rani Laxmibai College Parola Distt. Jalgaon (Maharashtra). He has been teaching mathematics subjects for degree classes for th past 25 years in Mahavidyalaya. He is Director, Research Laboratory of Mathematics subject and guiding students for PhD degree. He is also a member of Research Advisor Committee (RAC) at Pujya Sane Guruji ASC College Shahada, Dadasaheb Rawal ASC College Dondaicha, Pratap Mahavidyalaya Amalner and R.L. Mahavidyalaya Parola. He is a member of Board of studies in mathematics at KBC North Maharashtra University, Jalgaon since 2018.A total number of research papers 19 have been published in scoups, ugc care listed and peer reviewed journals. The main objective of writing this book is to make it useful for students and engineers doing research work in mathematics.