An Evaluation of the Dijkstra's Algorithm, Floyd's Algorithm and Ant Colony Optimization

A. J. Khan, Nikita Dewangan

***Abstract**: Travel is the movement of people between distant geographical locations. It can assist in enhancing our well-being, increasing our understanding, socializing with new individuals, relaxing and unwinding, seeking adventure, creating memories, improving mental and physical health, and immersing ourselves in different cultures. In this paper, we briefly explain the shortest path and their types applicable in practical life. The shortest path problem is a flexible and crucial instrument in different areas, facilitating effective route planning, network optimization, and resource distribution. There are many popular algorithms for solving the shortest distance path problem and its variations. We discuss how to create the most efficient route and introduce Dijkstra's algorithm, Floyd's algorithm, and Ant Colony Optimization to decrease the overall path expense, which could be distance, time, or another factor for the given scenario. We use two scenarios and evaluate Dijkstra's algorithm, Floyd's algorithm, and Ant Colony Optimization to determine the shortest route in practical situations to enhance efficiency in solving the identical issue.*

***Keywords:** Ant Colony Optimization, Dijkstra's algorithm, Floyd's algorithm, Shortest Path, Travelling.*

Abbreviations:
SPP: Shortest Path Problem
DA: Dijkstra's Algorithm
FWA: Floyd-Warshall Algorithm
ACO: Ant Colony Optimization

I. INTRODUCTION

A. Meaning of Path

We are accustomed to exploring various locations on a daily basis. We journey from our residences to educational institutions, universities, markets, and workplaces. Picture a system where all locations are linked by roadways and walkways, with each location labeled as a node, and the roadways and walkways serving as edges. A path consists of a series of edges connecting a starting node to an ending node.

In mathematics, individuals operate based on the pass length and set up the lowest average.

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The most direct route is the path connecting the two nodes with the shortest edges of equal costs for traversal. There exist numerous shortest paths between two points in a network due to the presence of various paths that share the same minimum cost, leading us to concentrate on the discussion of the shortest path between two points as opposed to the shortest path throughout the network.

For instance, when traveling from your residence to your educational institution in a different city, it can be challenging to determine the quickest route if there is a significant distance between the two places. Just like GPS applications like Google Maps and Apple Maps, how do you decide on the most optimal route to reach your destination? An alternative approach is to address the issue of shortest and quickest route by identifying the most economical route between two points [1].

B. Shortest Path Problem (SPP)

- The SPP is the process of locating the shortest route between vertices in a specified graph.
- The least expensive path between two vertices is the one that costs the least compared to all other option.

C. Types of Shortest Path Problems (SPP):

i. *Single-pair SPP: -*

- This pertains to resolving the fastest route between two locations on a graph.
- A popular algorithm often applied to identify the shortest path joining two points is the A* search algorithm.

ii. *Single-source SPP: -*

- This issue entails determining the most concise and expedient route from one point to every other point within a graph.
- Dijkstra's and Bellman-Ford's algorithms are commonly used to solve this type of problem.

iii. *Single-destination SPP: -*

- This pertains to identifying the shortest and most expedient route from each vertex to the singular designated final vertex.
- Changing the direction of the edges in the graph transforms the problem into one of determining the most efficient path from a single starting point.
- The Dijkstra algorithms are famous algorithms recognized for efficiently solving problems in the shortest possible way.

iv. *All pairs SPP: -*

- This pertains to determining the shortest and quickest path within each couple of vertexes.
- The Floyd-Warshall and Johnson algorithms both are widely noted algorithms for resolving the shortest path problem

for all couples of vertices.

D. Dijkstra's Algorithm

- Dijkstra's algorithm was presented in 1959 and is named after its inventor, Computer scientist Edsger Dijkstra.
- It is a general algorithm for identifying the shortest path between distinct sources with nonnegative edge weights in a weighted graph.
- Works on both Rieded and illegal graphics [2].

E. Steps of Dijkstra's algorithm

Step I. Initialize: Let O = {s} and Y = B-{s}. Let l (y) = r (s, y) for each y of Y.

Step II. Choose the vertex in Y with the lowest index relative to O. Label this vertex as z.

Step III. When z is a vertex, our goal is to determine its minimum distance from a stop. Alternatively, O' equals O without z and Y' equals Y excluding z. Calculate the relative index of each vertex y in Y' with respect to O' during the iteration process.

$$l'(y)=\min[l(y), l(z) + e(z, y)].$$

Step4. Repeat step I and II using O' as O and Y' as Y [3].

F. Floyd-Warshall Algorithm

I. The Floyd-Warshall algorithm was presented by Robert Floyd in 1962.
II. The algorithm is alternatively called as the Floyd algorithm, Roy-Warshall algorithm, Roy-Floyd algorithm, WFI algorithm.
III. It's utilized to ascertain the most abbreviated route between each pair of vertices within a weighted graph [4].
IV. It's employed to resolve the matter of identifying t he most effective path between each pair.
V. The calendar computes the most efficient route between all pairs of dates in the calendar.
VI. This is a demonstration of a method using dynamic programming [5].

G. Steps of Floyd-Warshall Algorithm

Step1. Form a square matrix F0 with size b*b, here b represents the number of vertices, and p and q correspond to the rows and columns denoting the vertices of the graph.

The length between the pth vertex and the qth vertex is stored in each cell F [p, q]. If no pathway exists from the pth vertex to the qth vertex, the cell is deemed infinite.

Step2. involves the creation of matrix F1 from matrix F0. The elements in the first column are identical to those in the first row. If F[p, q] > F[p, j] + F[j, q], then update F1[p, q] with the sum of F[p, j] and F[j, q] for the remaining values. Keep the same language and word count when paraphrasing the given text.

Currently, I am on stage J = 1, which is the initial peak in the shortest path from start to finish, serving as a pivotal point.

Step3. involves iterating through all the vertices in the graph, and updating the value of j for each reference vertex until the final matrix is obtained. Fj [p, q] equals the lowest value between Fj [p, q] = min [Fj-1[p, q], Fj-1[p, j] + Fj-1[j, q]]

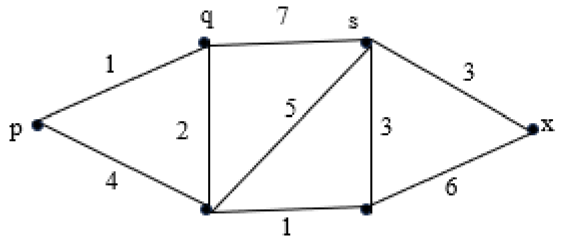
Step4. The ultimate solution with all shortest paths is represented by the final adjacency matrix reached [6].

H. Ant Colony Optimization

- Macro Dorigo from Italy created the Ant system during his doctoral thesis in 1992.
- Gambardella Dorigo created the Ant Colony in 1997.
- Ants move from their shelter to find food. They cannot see [7]!
- Each Ant uses pheromone trails to locate the shortest route, with every Ant moving randomly and leaving pheromones as they travel [8].
- Ants walk randomly and deposit pheromones on their course [9].
- The greater the number of pheromones along a path, the higher the probability of following that path [10].

II. DISCUSSION

Example 1: Find the shortest distance from p to x by Dijkstra's algorithm, Floyd-Warshall algorithm and Ant Colony Optimization.



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Solution: -
Solution by Dijkstra's algorithm: -
Let H= (B, R) denote the graph.
Here B = {p, q, r, s, t, x}.
Step1. Take O1={p}, Y1= {q, r, s, t, x}.
Now l(q)=1, l(r)=4, l(s)= l(t)= l(x)= ∞
Here q ϵ Y1 has the minimum index 1.
Step2. Taking O2= {p, q}, Y2= {r, s, t, x}.
Now l(r)= min (4, 1+2) = 3, l(s)= min (∞, 1+7) =8, l(t)= min (∞, 1+5) =6, l(x)= min (∞, 1+∞)= ∞.
Here r ϵ Y2 has the minimum index 3.
Step3. Taking O3= {p, q, r}, Y3= {s, t, x}.
Now l(s)= min (8, 3+∞) =8, l(t)= min (6, 3+1) =4, l(x)= min (∞, 3+∞)= ∞.
Here t ϵ Y3 has the minimum index 4.
Step4. Taking O4= {p, q, r, t}, T4= {s, x}.
Now l(s)= min (8, 4+3) =7, l(x)= min (∞, 4+6) =10.
Here s ϵ Y4 has the minimum index 7.
Step5. Taking O5= {p, q, r, t, s}, Y5= {x}.
Now l(x)=min (10+7+3) =10.
Simply,

From p	q	r	s	t	x
Step1	1*	4	∞	∞	∞
Step2		3*	7	5	∞
Step3			7	4*	∞
Step4			7*		10
Step5					10*

Hence shortest path from p to x is p→ q→ r→ t→ s→ x and the length is 10.

Solution by Floyd-Warshall algorithm: -
Step1. Construction of a matrix F0 of dimension 6×6.
Substituting the values in the following way:
Fj [p, q] =min [Fj-1[p, q],
Fj-1[p, j] + Fj-1[j, q]]

$$F^0=\begin{bmatrix} 0 & 1 & 4 & \infty & \infty & \infty \\ 1 & 0 & 2 & 7 & 5 & \infty \\ 4 & 2 & 0 & \infty & 1 & \infty \\ \infty & 7 & \infty & 0 & 3 & 3 \\ \infty & 5 & 1 & 3 & 0 & 6 \\ \infty & \infty & \infty & 3 & 6 & 0 \end{bmatrix}$$

$$F^1=\begin{bmatrix} 0 & 1 & 4 & \infty & \infty & \infty \\ 1 & 0 & 2 & 7 & 5 & \infty \\ 4 & 2 & 0 & \infty & 1 & \infty \\ \infty & 7 & \infty & 0 & 3 & 3 \\ \infty & 5 & 1 & 3 & 0 & 6 \\ \infty & \infty & \infty & 3 & 6 & 0 \end{bmatrix}$$

$$F^2=\begin{bmatrix} 0 & 1 & 3 & 8 & 6 & \infty \\ 1 & 0 & 2 & 7 & 5 & \infty \\ 3 & 2 & 0 & 9 & 1 & \infty \\ 8 & 7 & 9 & 0 & 3 & 3 \\ 6 & 5 & 1 & 3 & 0 & 6 \\ \infty & \infty & \infty & 3 & 6 & 0 \end{bmatrix}$$

$$F^3=\begin{bmatrix} 0 & 1 & 3 & 8 & 4 & \infty \\ 1 & 0 & 2 & 7 & 3 & \infty \\ 3 & 2 & 0 & 9 & 1 & \infty \\ 8 & 7 & 9 & 0 & 3 & 3 \\ 4 & 3 & 1 & 3 & 0 & 6 \\ \infty & \infty & \infty & 3 & 6 & 0 \end{bmatrix}$$

$$F^4=\begin{bmatrix} 0 & 1 & 3 & 8 & 4 & 11 \\ 1 & 0 & 2 & 7 & 3 & 10 \\ 3 & 4 & 0 & 9 & 1 & 12 \\ 8 & 7 & 9 & 0 & 3 & 3 \\ 4 & 3 & 1 & 3 & 0 & 6 \\ 11 & 10 & 12 & 3 & 6 & 0 \end{bmatrix}$$

Hence shortest path from p to x is p→ q→ r→ t→ s→ x and the length is 1+2+1+3+3=10.

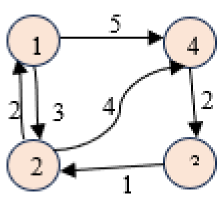
Solution by Ant Colony Optimization: -

Ant starts travelling from node p and has to reach node x. Followings will be multiple paths to reach node x.

1. p→ q→ r→ t→ s→ x
   Total distance covered: 1+2+1+3+3 = 10
2. p → r → q → s → t → x
   Total distance covered: 4+2+7+3+6 = 22
3. p → r → q → t → s → x
   Total distance covered: 4+2+5+3+3 = 17
4. p → q → s → x
   Total distance covered: 1+7+3 = 11
5. p → q → t → x
   Total distance covered: 1+5+6 = 12

By comparing these multiple paths, Pheromones in paths 2, 3, 4, and 5 will quickly evaporate as compared to path 1. Since the total covered distance is more than Path 1. Because of more pheromones, Ant will choose the first path p → q → r → t → s → x. Hence shortest path from p to x is Path 1.

Example 2: Find the shortest distance from to cover/travel all vertex by Dijkstra's algorithm as well as Floyd-Warshall algorithm.



Let H= (B, R) denote the graph. Here B = {1,2,3,4}.

Solution by Dijkstra's algorithm: -

In directed graph we get many paths with different weight. If we take P1= {1}, Y1= {2,3,4}.

From 1	2	3	4
Step1	3*	∞	5
Step2		∞	7*
Step3		9*	

Here, shortest path to cover all vertex is 1→2→4→3 and length is 3+4+2=9. If we start from 2,

From 2	1	3	4
Step1	2*	∞	4
Step2		∞	7*
Step3		9*	

shortest path to cover all vertex is 2→1→4→3 and length is 2+5+2=9. If we start from 3,

From 3	1	2	4
Step1	∞	1*	∞
Step2	3*		5
Step3		8*	

shortest path to cover all vertex is 3→2→1→4 and length is 1+2+5=8. If we start from 4,

From 4	1	2	3
Step1	∞	∞	2*
Step2	∞	3*	
Step3	5*		

The shortest path to cover all vertex is 4→3→2→1 and length is 2+1+2=5. Here we get 4 different paths, if we choose different starting vertex.

Hence shortest path to cover all vertex is 4→3→2→1 and length is 5.

Solution by Floyd-Warshall algorithm:

Step1. Construction of a matrix F0 of dimension 4×4.

Substituting the values by the following way:

Fj [p, q] =min [Fj-1[p, q], Fj-1[p, j] + Fj-1[j, q]]

$$F0=\begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & \infty & 4 \\ \infty & 1 & 0 & \infty \\ \infty & \infty & 2 & 0 \end{bmatrix}$$

$$F1=\begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & \infty & 4 \\ \infty & 1 & 0 & \infty \\ \infty & \infty & 2 & 0 \end{bmatrix}$$

$$F2=\begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & \infty & 4 \\ 3 & 1 & 0 & 5 \\ \infty & \infty & 2 & 0 \end{bmatrix}$$

$$F^3=\begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & \infty & 4 \\ 3 & 1 & 0 & 5 \\ 5 & 3 & 2 & 0 \end{bmatrix}$$

$$F^4=\begin{bmatrix} 0 & 3 & 7 & 5 \\ 2 & 0 & 6 & 4 \\ 3 & 1 & 0 & 5 \\ 5 & 3 & 2 & 0 \end{bmatrix}$$

Here we can see that we can find all distance easily, like if I want to know distance from 1 to 4 then we see 1st row 4th column, this is 5. Similarly, from 3 to 2, we see 3rd row 2nd column, etc.

Solution by Ant Colony Optimization: -

Ant start travelling from any node and has to cover all nodes i.e.1,2,3,4. Followings will be multiple paths.

Path1: 1 → 4 → 3 → 2 Total distances covered 5+2+1 = 8
Path2: 1 → 2 → 4 → 3 Total distances covered 3+4+2= 9
Path3: 2 → 1 → 4 → 3 Total distances covered 2+5+2 = 9
Path4: 3 → 2 → 1 → 4 Total distances covered 1+2+5 = 8
Path5: 4 → 3 → 2 → 1 Total distances covered 2+1+2 = 5

By comparing these multiple paths, Pheromones in paths 1, 2, 3, and 4 will quickly evaporate as compared to path 5. Since the total covered distance is more than path 5. Because of more pheromones, Ant will choose first path 4→3→2→ 1. Hence quickest path to cover all the vertices is Path 5.

A. Dijkstra's Algorithm (DA) VS Floyd-Warshall Algorithm (FWA) VS Ant Colony Optimization (ACO)

- DA finds the shortest pathways between vertices in a weighted graph. One method used to determine the most effective paths between every couple of points in a weighted graph is the FWA. ACO is a metaheuristic approach that is grounded in the behavior of ants.
- DA is considered a Greedy approach because it chooses the vertex with the smallest distance. The FWA employs Dynamic programming to identify the shortest paths within every couple of points. ACO is a food searching approach by Ant which is based on pheromone trail.
- Negative edge weights can cause DA to malfunction. Both graphs with positive and negative edge weights can be handled using the FWA. ACO follows the high-density pheromone path.
- In DA, a time difficulty of with a priority queue or min-heap I(R + B*log(B)). The difficulty relating to the time of the FWA is I(B3). Where R= no. of the edges and B= no. of the vertices. ACO is a forging behavior of Ants and it needs more inputs to come to a single conclusion.

III. CONCLUSION

The shortest path problem continues to be among the top subjects studied in the area. This text presents the fundamental concept of three shortest-path algorithms - Dijkstra's algorithm, Floyd-Warshall Algorithm, and Ant Colony Optimization. And compares them based on the same problems. To summarize, this paper has investigated three important shortest path algorithms. We also compared them using significant criteria. Only one precise beginning point can be utilized to determine the shortest path using the Dijkstra algorithm. However, the Floyd-Warshall algorithm can be utilized to find the most efficient path between any pair of destinations. It is adept at finding the shortest path amongst all vertices or a small set of data. The concept of Ant Colony Optimization is based on how ants move from their shelter to a food supply. To sum up, The Dijkstra algorithm is used for sparse graphs. In practice, the algorithm is constantly fine-tuned, such as with heap Optimization. Pathfinding is not ideal for Ant Colony Optimization due to its instability and lengthy search process. The Floyd-Warshall algorithm is the least efficient and consumes excessive memory space when handling a significant number of points and edges.

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

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- **Authors Contributions:** The authorship of this article is contributed equally to all participating individuals.

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Dr. A. J. Khan, working as Principal and Associate Professor at MATS College, MATS University, Raipur having experience of 25 years in teaching experience at college level. He has been working at MATS college as Professor since last 12 years. Earlier to that he has served 6 years at Disha Institute of Management and Technology, Raipur. He has published many research papers which includes Operational research topics like Genetic Algorithm, Ant colony Optimization, many topics related to finding shortest distance, etc. His future aspect is to grow his domain in mathematics by doing research which can be useful for researchers and scholars.

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