An Elementary Proof for Fermat's Last Theorem using Ramanujan-Nagell Equation

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***Abstract:** Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation $A^n + B^n = C^n$ where n is any integer > 2. Taking the proofs of Fermat for the index n = 4, and Euler for n = 3, it is sufficient to prove the theorem for n = p, any prime > 3. We hypothesize that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$ and establish a contradiction in this proof. Just for supporting the proof in the above equation, we have used another equation $x^3 + y^3 = z^3$ Without loss of generality, we assert that both x and y as non-zero integers; z^3 a non-zero integer; z and z^2 irrational. We create transformed equations to the above two equations through parameters, into which we have incorporated the Ramanujan - Nagell equation. Solving the transformed equations we prove the theorem.*

***Keywords:** Transformed Fermat's Equations through Parameters. 2010 Mathematics Subject Classification 2010: 11A–XX.*

I. INTRODUCTION

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of a book that the equation $A^n + B^n = C^n$ has no solution in integers *A*, *B* and *C*, if n is any integer >2. Fermat stated in the margin of the book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index n=4, using infinite descent method [1].

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory [2], [3], [3][4][5][6][7][8][9]. Here we are trying an elementary proof.

II. ASSUMPTIONS

1) We initially hypothesize that all *r*, *s* and *t* are non-zero integers satisfying the equation $r^p + s^p = t^p$ where *p* is any prime > 3, with gcd(*r*, *s*, *t*) = 1 and establish a contradiction in this proof.
2) Just for supporting the proof in the above equation, we have taken another equation.

$$x^3 + y^3 = z^3; \qquad \gcd(x, y, z^3) = 1$$

Without loss of generality, we can have both *x* and *y* as non-zero integers, z^3 a non-zero integer; both *z* and z^2 irrational. Since we prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of *r*, *s* and *t* we have the choice in having *x*=17; *y*=20; $z^3 = 17^3+20^3=37\times349$ and so on, such that the odd prime factors in *x*, *y* and z^3 could be chosen as coprimes to each of *r*, *s* and *t*. In the transformation equation the pattern and structures are to be maintained whatever the odd prime factors of *x*, *y* and z^3 are used inside the square roots.

3) We have used the Ramanujan-Nagell equation solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$ in $2^n = 7 + \ell^2$, where *n* is odd and $\ell > 1$.
4) In this proof we assign the values as *x* = 11, *y* = 53, $z^3 = 11^3 + 53^3 = 8^2 \times 2347$.
5) Let *E* and *R* be distinct odd primes each coprime to each of *x*, *y*, z^3, *r*. *s*, *t*, 7 and ℓ; *F* = (2*y*) = 2 × 53.

Proof. By random trials, we have created the following equations,

$$\left(a\sqrt{s}+b\sqrt{\ell^{5/3}}\right)^2+\left(c\sqrt{7^{7/3}}+d\sqrt{E^{5/3}}\right)^2=\left(\frac{e\sqrt{F^{1/3}}+f\sqrt{E^{1/3}}}{\sqrt{7^{5/3}}}\right)^2$$

and

$$\left(a\sqrt{2^{n/2}}-b\sqrt{F^{1/3}}\right)^2+\left(\frac{c\sqrt{t}-d\sqrt{R^{1/3}}}{\sqrt{2^{3n/2}}}\right)^2=\left(\frac{e\sqrt{r}-f\sqrt{R^{5/3}}}{\sqrt{\ell^{7/3}}}\right)^2 \qquad (1)$$

as the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively through the parameters called *a*, *b*, *c*, *d*, *e* and *f*. Here we have assigned the values *x*= 11, *y* = 53, $z^3 = 11^3 + 53^3 = 8^2 \times 2347$. *E* and *R* are distinct odd primes, each is coprime to *x*, *y*, z^3, *r*, *s*, *t*, 7 and ℓ, where $2^n = 7 + \ell^2$, where *n* is odd and $\ell > 1$ using one of the solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$ and *F* = (2*y*)

From equation (1), we get

$$a\sqrt{s} + b\sqrt{\ell^{5/3}} = \sqrt{x^3} \qquad (2)$$

$$a\sqrt{2^{n/2}} - b\sqrt{F^{1/3}} = \sqrt{r^p} \qquad (3)$$

$$c\sqrt{7^{1/3}} + d\sqrt{E^{5/3}} = \sqrt{y^3} \qquad (4)$$

$$c\sqrt{t} - d\sqrt{R^{1/3}} = \sqrt{2^{3n/2}s^p} \qquad (5)$$

$$e\sqrt{F^{1/3}} + f\sqrt{E^{1/3}} = \sqrt{7^{5/3}z^3} \qquad (6)$$

And $$e\sqrt{r} - f\sqrt{R^{5/3}} = \sqrt{\ell^{7/3}t^p} \qquad (7)$$

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Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a = \left(\sqrt{F^{1/3}x^3} + \sqrt{r^p \ell^{5/3}}\right) \Big/ \left(\sqrt{F^{1/3}s} + \sqrt{2^{n/2} \times \ell^{5/3}}\right)$$

$$b = \left(\sqrt{2^{n/2}x^3} - \sqrt{r^p s}\right) \Big/ \left(\sqrt{F^{1/3}s} + \sqrt{2^{n/2} \times \ell^{5/3}}\right)$$

$$c = \left(\sqrt{R^{1/3}y^3} + \sqrt{2^{3n/2}E^{5/3}s^p}\right) \Big/ \left(\sqrt{(7R)^{1/3}} + \sqrt{E^{5/3}t}\right)$$

$$d = \left(\sqrt{y^3 t} - \sqrt{2^{3n/2}7^{1/3}s^p}\right) \Big/ \left(\sqrt{(7R)^{1/3}} + \sqrt{E^{5/3}t}\right)$$

$$e = \left(\sqrt{(7R)^{5/3}z^3} + \sqrt{E^{1/3}\ell^{7/3}t^p}\right) \Big/ \left(\sqrt{F^{1/3}R^{5/3}} + \sqrt{E^{1/3}r}\right)$$

and $$f = \left(\sqrt{7^{5/3}z^3 r} - \sqrt{F^{1/3}\ell^{7/3}t^p}\right) \Big/ \left(\sqrt{F^{1/3}R^{5/3}} + \sqrt{E^{1/3}r}\right)$$

From (3) & (5), we have

$$\sqrt{2^{n/2}} \times \sqrt{2^{3n/2}} = \left(\sqrt{r^p} + b\sqrt{F^{1/3}}\right)\left(c\sqrt{t} - d\sqrt{R^{1/3}}\right) \Big/ \left(a\sqrt{s^p}\right)$$

i.e., $$2^n = \left\{(c)\sqrt{r^p t} - (d)\sqrt{R^{1/3}r^p} + (bc)\sqrt{F^{1/3}t} - (bd)\sqrt{(FR)^{1/3}}\right\} \Big/ \left(a\sqrt{s^p}\right)$$

From (4) & (6), we get

$$\sqrt{7^{1/3}} \times \sqrt{7^{5/3}} = \left(\sqrt{y^3} - d\sqrt{E^{5/3}}\right)\left(e\sqrt{F^{1/3}} + f\sqrt{E^{1/3}}\right) \Big/ \left(c\sqrt{z^3}\right)$$

i.e., $$7 = \left\{(e)\sqrt{F^{1/3}y^3} + (f)\sqrt{E^{1/3}y^3} - (de)\sqrt{F^{1/3}E^{5/3}} - (E)(df)\right\} \Big/ \left(c\sqrt{z^3}\right)$$

From (2) & (7), we get

$$\sqrt{\ell^{7/3}} \times \sqrt{\ell^{5/3}} = \left(\sqrt{x^3} - a\sqrt{s}\right)\left(e\sqrt{r} - f\sqrt{R^{5/3}}\right) \Big/ \left(b\sqrt{t^p}\right)$$

i.e., $$\ell^2 = \left\{(e)\sqrt{x^3 r} - (f)\sqrt{R^{5/3}x^3} - (ae)\sqrt{rs} + (af)\sqrt{R^{5/3}s}\right\} \Big/ \left(b\sqrt{t^p}\right)$$

Substituting the above equivalent values in Ramanujan-Nagell equation $2^n = 7 + \ell^2$ after multiplying both sides by $\left\{(abc)\sqrt{z^3 s^p t^p}\right\}$, we get

$$\left\{(bc)\sqrt{z^3 t^p}\right\}\left\{(c)\sqrt{r^p t} - (d)\sqrt{R^{1/3}r^p} + (bc)\sqrt{F^{1/3}t} - (bd)\sqrt{(FR)^{1/3}}\right\}$$
$$= \left\{(ab)\sqrt{s^p t^p}\right\}\left\{(e)\sqrt{F^{1/3}y^3} + (f)\sqrt{E^{1/3}y^3} - (de)\sqrt{F^{1/3}E^{5/3}} - (df)(E)\right\}$$
$$+ \left\{(ac)\sqrt{z^3 s^p}\right\}\left\{(e)\sqrt{x^3 r} - (f)\sqrt{R^{5/3}x^3} - (ae)\sqrt{rs} + (af)\sqrt{R^{5/3}s}\right\} \quad (8)$$

Let us find out all rational terms in equation (8) after multiplying both sides by

$$\left\{\left(\sqrt{F^{1/3}s} + \sqrt{2^{n/2}\ell^{5/3}}\right)^2 \left(\sqrt{(7R)^{1/3}} + \sqrt{E^{1/3}t}\right)^2 \left(\sqrt{F^{1/3}R^{5/3}} + \sqrt{E^{1/3}r}\right)\right\}$$

to be free from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* and again multiplying both sides by $\left\{\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\right\}$ for getting some rational terms as worked out hereunder, term by term.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{bc^2\}$

$$= \sqrt{t^{p+1}}\sqrt{z^3 r^p}\left(\sqrt{F^{1/3}s} + \sqrt{2^{n/2}\ell^{5/3}}\right)\left(\sqrt{F^{1/3}R^{5/3}} + \sqrt{E^{1/3}r}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}$$
$$\times\left(\sqrt{2^{n/2}x^3} - \sqrt{r^p s}\right)\left\{\left(R^{1/3}y^3\right) + \left(2\sqrt{R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right) + \left(E^{5/3}s^p\sqrt{2^{3n}}\right)\right\}$$

On multiplying by

$$\left\{\sqrt{t^{p+1}}\sqrt{z^3 r^p}\sqrt{F^{1/3}s}\sqrt{F^{1/3}R^{5/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\left(-\sqrt{r^p s}\right)\left(2\sqrt{R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right)\right\}$$

We get

$$\left\{-\left(2^{n+1}ERr^p z^3 s\right)\sqrt{(st)^{p+1}}\sqrt{Fy^3}\right\}$$

This term gets cancelled with the term worked out under III term in LHS below.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)\}$

$$=\left(-\sqrt{R^{1/3}z^3r^pt^p}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}}\right)\left(\sqrt{F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}$$
$$\times\left(\sqrt{2^{n/2}x^3}-\sqrt{r^ps}\right)\left(\sqrt{R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{y^3t}-\sqrt{2^{3n/2}7^{1/3}s^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{R^{1/3}z^3r^pt^p}\right)\sqrt{F^{1/3}s}\sqrt{F^{1/3}R^{5/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left(-\sqrt{r^ps}\right)\sqrt{2^{3n/2}E^{5/3}s^p}\sqrt{y^3t}\right\}$$

We get

$$\left\{\left(2^nERr^pz^3s\right)\sqrt{(st)^{p+1}}\sqrt{Fy^3}\right\}$$

This term gets cancelled with the term worked out under IV term in LHS below.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2c^2\}$

$$=\sqrt{t^{p+1}}\sqrt{F^{1/3}z^3}\left(\sqrt{F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left\{\left(x^3\sqrt{2^n}\right)+\left(r^ps\right)-2\sqrt{2^{n/2}x^3r^ps}\right\}$$
$$\times\left\{\left(R^{1/3}y^3\right)+\left(2\sqrt{R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right)+\left(E^{5/3}s^p\sqrt{2^{3n}}\right)\right\}$$

On multiplying by

$$\left\{\sqrt{t^{p+1}}\sqrt{F^{1/3}z^3}\sqrt{F^{1/3}R^{5/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left(r^ps\right)\left(2\sqrt{R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right)\right\}$$

We get

$$\left\{\left(2^{n+1}ERr^pz^3s\right)\sqrt{(st)^{p+1}}\sqrt{Fy^3}\right\}$$

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2(cd)\}$

$$=\left(-\sqrt{z^3t^p}\sqrt{F^{1/3}R^{1/3}}\right)\left(\sqrt{F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left(\sqrt{R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)$$
$$\times\left\{\left(x^3\sqrt{2^n}\right)+\left(r^ps\right)-2\sqrt{2^{n/2}x^3r^ps}\right\}\left(\sqrt{y^3t}-\sqrt{2^{3n/2}7^{1/3}s^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{z^3t^p}\right)\sqrt{F^{1/3}R^{1/3}}\sqrt{F^{1/3}R^{5/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\sqrt{2^{3n/2}E^{5/3}s^p}\left(r^ps\right)\sqrt{y^3t}\right\}$$

We get

$$\left\{-\left(2^nERr^pz^3s\right)\sqrt{(st)^{p+1}}\sqrt{Fy^3}\right\}$$

This term gets cancelled with II term in LHS in equation (8).

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)e\}$

$$=\sqrt{F^{1/3}y^3s^pt^p}\left\{(7R)^{1/3}+\left(E^{5/3}t\right)+2\sqrt{(7R)^{1/3}}\sqrt{E^{5/3}t}\right\}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}$$
$$\times\left(\sqrt{2^{n/2}x^3}-\sqrt{r^ps}\right)\left(\sqrt{F^{1/3}x^3}+\sqrt{\ell^{5/3}r^p}\right)\left(\sqrt{(7R)^{5/3}z^3}+\sqrt{E^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\sqrt{F^{1/3}y^3s^pt^p}\left(2\sqrt{(7R)^{1/3}}\sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\sqrt{F^{1/3}x^3}\sqrt{2^{n/2}x^3}\sqrt{(7R)^{5/3}z^3}\right\}$$

We get

$$\left\{\left(2\times7ERx^3z^3\right)\sqrt{(st)^{p+1}}\sqrt{F\times2^ny^3}\right\}$$

Which will be rational, since we have $F = 2y$ and n is odd integer.

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II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)f\}$

$$= \sqrt{E^{1/3}y^3 s^p t^p}\left\{(7R)^{1/3} + \left(E^{5/3}t\right) + 2\sqrt{(7R)^{1/3}}\sqrt{E^{5/3}t}\right\}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}$$
$$\times\left(\sqrt{2^{n/2}x^3} - \sqrt{r^p s}\right)\left(\sqrt{F^{1/3}x^3} + \sqrt{\ell^{5/3}r^p}\right)\left(\sqrt{7^{5/3}z^3 r} - \sqrt{F^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\sqrt{E^{1/3}y^3 s^p t^p}\left(E^{5/3}t\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\sqrt{2^{n/2}x^3}\sqrt{F^{1/3}x^3}\left(-\sqrt{F^{1/3}\ell^{7/3}t^p}\right)\right\}$$

We get

$$\left\{-\left(E^2x^3\right)t^{p+1}\sqrt{s^{p+1}}\sqrt{F\times 2^n y^3 z^3 \ell^{7/3}}\right\}$$

Which will be irrational since $F = (2y)$.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{abde\}$

$$= \left(-\sqrt{F^{1/3}E^{5/3}s^p t^p}\right)\left(\sqrt{(7R)^{1/3}} + \sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\left(\sqrt{F^{1/3}x^3} + \sqrt{\ell^{5/3}r^p}\right)$$
$$\times\left(\sqrt{2^{n/2}x^3} - \sqrt{r^p s}\right)\left(\sqrt{y^3 t} - \sqrt{2^{3n/2}7^{1/3}s^p}\right)\left(\sqrt{(7R)^{5/3}z^3} + \sqrt{E^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{F^{1/3}E^{5/3}s^p t^p}\right)\sqrt{(7R)^{1/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\sqrt{F^{1/3}x^3}\sqrt{2^{n/2}x^3}\sqrt{y^3 t}\sqrt{(7R)^{5/3}z^3}\right\}$$

We get

$$\left\{-\left(7ERx^3z^3\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3}\right\}$$

Which will be rational.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)df\}$

$$= \left(-E\sqrt{s^p t^p}\right)\left(\sqrt{(7R)^{1/3}} + \sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\left(\sqrt{F^{1/3}x^3} + \sqrt{\ell^{5/3}r^p}\right)$$
$$\times\left(\sqrt{2^{n/2}x^3} - \sqrt{r^p s}\right)\left(\sqrt{y^3 t} - \sqrt{2^{3n/2}7^{1/3}s^p}\right)\left(\sqrt{7^{5/3}z^3 r} - \sqrt{F^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-E\sqrt{s^p t^p}\right)\sqrt{E^{5/3}t}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\sqrt{F^{1/3}x^3}\sqrt{2^{n/2}x^3}\sqrt{y^3 t}\left(-\sqrt{F^{1/3}\ell^{7/3}t^p}\right)\right\}$$

We get

$$\left\{\left(E^2x^3t^{p+1}\right)\sqrt{s^{p+1}}\sqrt{F\times 2^n y^3 z^3 \ell^{7/3}}\right\}$$

Which will be irrational, moreover this term gets cancelled with II term in RHS.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ace\}$

$$= \sqrt{x^3z^3rs^p}\left(\sqrt{F^{1/3}s} + \sqrt{2^{n/2}\ell^{5/3}}\right)\left(\sqrt{(7R)^{1/3}} + \sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\left(\sqrt{F^{1/3}x^3} + \sqrt{\ell^{5/3}r^p}\right)$$
$$\times\left(\sqrt{R^{1/3}y^3} + \sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{(7R)^{5/3}z^3} + \sqrt{E^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\sqrt{x^3z^3rs^p}\sqrt{F^{1/3}s}\sqrt{(7R)^{1/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3 s}\sqrt{F^{1/3}x^3}\sqrt{2^{3n/2}E^{5/3}s^p}\sqrt{(7R)^{5/3}z^3}\right\}$$

We get

$$\left\{\left(2^n\times 7ERx^3z^3s^{p+1}\right)\sqrt{Frz^3}\right\}$$

Which is irrational, since *we* have $F = 2y$; r is coprime to $y = 53$; and 2347.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{def\}$ is

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$$= \left(-\sqrt{R^{5/3}x^3z^3s^p}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}}\right)\left(\sqrt{(7R)^{1/3}}+\sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}$$
$$\times\left(\sqrt{F^{1/3}x^3}+\sqrt{\ell^{5/3}r^p}\right)\left(\sqrt{R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{7^{5/3}z^3r}-\sqrt{F^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{R^{5/3}x^3z^3s^p}\right)\sqrt{2^{n/2}\ell^{5/3}}\sqrt{E^{5/3}t}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\sqrt{F^{1/3}x^3}\sqrt{R^{1/3}y^3}\left(-\sqrt{F^{1/3}\ell^{7/3}t^p}\right)\right\}$$

We get

$$\left\{\left(ERx^3z^3\ell^2\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3}\right\}$$

Which is rational, since we have $F = 2y$; n being odd.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2ce)\}$ is

$$= \left(-\sqrt{s^{p+1}}\sqrt{z^3r}\right)\left(\sqrt{(7R)^{1/3}}+\sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left\{\left(F^{1/3}x^3\right)+\left(\ell^{5/3}r^p\right)+2\sqrt{F^{1/3}r^p\ell^{5/3}x^3}\right\}$$
$$\times\left(\sqrt{R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{(7R)^{5/3}z^3}+\sqrt{E^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{s^{p+1}}\sqrt{z^3r}\right)\sqrt{(7R)^{1/3}}\left(x^3\sqrt{F^{2/3}}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\sqrt{2^{3n/2}E^{5/3}s^p}\sqrt{(7R)^{5/3}z^3}\right\}$$

We get

$$\left\{-\left(2^n\times 7ERx^3z^3s^{p+1}\right)\sqrt{Fz^3r}\right\}$$

Which will be irrational, since $F = 2y$ and r is coprime to $y = 53$, and 2347, where $z^3 = 8^2 \times 2347$, otherwise we have the choice of assigning some other values for x, y, z^3, such that r is coprime to odd prime factors in x, y and z^3.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2cf)\}$ is

$$= \sqrt{s^{p+1}}\sqrt{R^{5/3}z^3}\left(\sqrt{(7R)^{1/3}}+\sqrt{E^{5/3}t}\right)\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left\{\left(x^3\sqrt{F^{2/3}}\right)+\left(\ell^{5/3}r^p\right)+2\sqrt{F^{1/3}r^p\ell^{5/3}x^3}\right\}$$
$$\times\left(\sqrt{R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{7^{5/3}z^3r}+\sqrt{F^{1/3}\ell^{7/3}t^p}\right)$$

On multiplying by

$$\left\{\sqrt{s^{p+1}}\sqrt{R^{5/3}z^3}\sqrt{7^{1/3}R^{1/3}}\sqrt{2^{n/2}F^{1/3}E^{1/3}z^3s}\left(x^3\sqrt{F^{2/3}}\right)\sqrt{2^{3n/2}E^{5/3}s^p}\sqrt{7^{5/3}z^3r}\right\}$$

We get

$$\left\{\left(2^n\times 7ERx^3z^3s^{p+1}\right)\sqrt{Fz^3r}\right\}$$

Which is irrational.

Sum of all rational part in LHS of equation (8) is NIL.

Sum of all rational part in RHS of equation (8) is

$$= \left(7ERx^3z^3\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3} \qquad \text{(comnining I \& III terms)}$$

$$+\left(ERx^3z^3\ell^2\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3} \qquad \text{(vide VI term)}$$

$$= \left(2^nERx^3z^3\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3} \qquad \left(\because\ 7+\ell^2 = 2^n\right)$$

Equating the rational terms on both sides of equation (8), we get

$$\left(2^nERx^3z^3\right)\sqrt{(st)^{p+1}}\sqrt{F\times 2^n y^3} = 0$$

Dividing both sides by

$$\left(2^nERx^3z^3\right)\sqrt{F\times 2^n y^3}$$

We get

$$\sqrt{(st)^{p+1}} = 0$$

That is, either $s = 0$; or $t = 0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$, with p any prime > 3, thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

Since equation (8) in this proof has been derived directly from the transformation equations the result that we have obtained on equating the rational terms on both sides of equation (8) should reflect on the Fermat's Equation $r^p + s^p = t^p$, thus proving that only a trivial solution exists in the equation $r^p + s^p = t^p$.

The only main hypothesis that we made in this proof, namely, r, s and t are non-zero integers has been shattered by the result $st = 0$, thus proving the theorem.

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