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On the Model of Transport of Salt Solution in a Xylem

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***Abstract:** We consider a model for vertical lifting of an aqueous solution of salts in a xylem. We analyzed changes of movement of the considered solution depending on various parameters. We consider an analytical approach for analysis of the considered model. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered processes.*

***Keywords:** Xylem; Transport of Aqueous Solution of Salts; Analytical Approach for Analysis*.

I. INTRODUCTION

Water is involved in ensuring the life of plants. It participates in formation of internal structure and appearance of the plant [1-5][10]. One of the purposes of xylem is transport nutrients from the soil through the roots to other plant organs. The main aim of the present paper is to formulate a model for the rise of an aqueous solution of salts (as nutrients for the plant) in the xylem and to analyze changes in the movement of the solution depends on several parameters. The approach gives a possibility to take into account changes of these parameters in space and time, as well as the nonlinearity of the considered processes.

II. METHOD OF SOLUTION

To solve the considered aims we determine the spatio-temporal distribution of concentration solution of salts in a xylem. We consider the above distribution of the concentration of salts as a solution of the second Fick's law in the following form [6]

$$\frac{\partial C(z,t)}{\partial t}+div\left[\vec{V}\cdot C(z,t)\right]=div\left\{D(T)\,grad\left[C(z,t)\right]\right\}-K(z,t)\,C(z,t)\cdots(1)$$

where C (z,t) is the concentration of the considered salts; $\vec{V}$ is the velocity of movement of the considered solution of salts with the single nonzero projection in z- direction; z is the axial coordinate; t is the current time; D is the diffusion coefficient of the salts in the considered solution; K is the parameter, characterizing the absorption of salts from solution by the considered plant.

The velocity of movement of the salt solution is described by the Navier- Stokes equation [6,7]

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\cdot\nabla\right)\vec{V}=-\nabla\left(\frac{P}{\rho}\right)+\nu\,\Delta\vec{V}\ \ldots..\ (2)$$

где P is the pressure in the considered xylem; ν is the kinematics viscosity; ρ is the density of solution. Boundary and initial conditions for the considered functions could be written as

$C(-L,t) = C_0$, $C(L,t) = 0$, $C(z,0) = C_0\delta(z+L)$, $V_z(-L,t) = V_0$, $V_z(L,t) = V_0$, $V_z(-L,0) = V_0$..... (3)

Equation for projection for considered velocity in cylindrical system of coordinate could be written as

$$\frac{\partial V_z}{\partial t}=\nu\frac{\partial^2 V_z(r,\varphi,z,t)}{\partial z^2}-V_z\frac{\partial V_z}{\partial z}-\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)..\ (4)$$

We solved the above equation by method of averaging of function corrections [6,8,9]. In the framework of the method to determine the first-order approximation of projection of the considered velocity of the solution of salts one shall replace it on the not yet known average value $V_z \to \alpha_{1z}$ in the right side of the equation (4). The substitution gives a possibility to obtain the following equation to determine the first-order approximation of the considered projection

$$\frac{\partial V_{1z}}{\partial t}=-\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)\ \ldots\ldots.\ (5)$$

Solution of the equation could be written as

$$V_{1z}=V_0-\frac{\partial}{\partial z}\int_0^t\frac{P}{\rho}\,d\tau\ \ldots\ldots.\ (6)$$

The second-order approximation of the considered projection could be obtained by replacing of the projection in the right side of the equation (4) on the following sum: $V_z \to \alpha_{2z}+V_{1z}$. The replacement gives a possibility to obtain the following equation to determine the required approximation in the following form

$$\frac{\partial V_{2z}}{\partial t}=-\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)-\left(\alpha_{2z}+V_{1z}\right)\frac{\partial V_{1z}}{\partial z}\ \ldots\ldots\ (7)$$

Integration of the obtained equation on time leads to the following result

$$V_{2z}=V_0+\nu\int_0^t\frac{\partial^2 V_{1z}}{\partial z^2}\,d\tau-\frac{\partial}{\partial z}\left(\int_0^t\frac{P}{\rho}\,d\tau\right)-\int_0^t\left(\alpha_{2z}+V_{1z}\right)\frac{\partial V_{1z}}{\partial z}\,d\tau\cdots8)$$

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The considered average value α_{2z} could be obtained by using the following standard relation

$$\alpha_{2z} = \frac{1}{\Theta L}\int_0^{\Theta}\int_{-L}^{L}\left(V_{2z} - V_{1z}\right) d\,z\,d\,t \ldots (9)$$

where Θ is the continuance of the considered process. Substitution of the first- and the second-order approximations in the relation (9) gives a possibility to obtain the following relation to determine the considered average value

$$\alpha_{2z} = \frac{1}{\Theta L}\int_0^{\Theta}(\Theta - t)\int_{-L}^{L}\left[\nu\frac{\partial^2 V_{1z}}{\partial z^2} - \left(\alpha_{2z} + V_{1z}\right)\frac{\partial V_{1z}}{\partial z}\right] d\,z\,d\,t \cdot (10)$$

We analyzed the considered projection of velocity of flow of solution of salts analytically by using the second-order approximations in the framework of the method of averaging of function corrections. Usually the second-order approximation is the enough good approximation to do qualitative analysis and to obtain quantitative results. All analytical results were checked by numerical simulation. Next we calculate the spatio-temporal distribution of concentration of the considered salts by solving of the equation (1) by the same method of averaging of function corrections. The first-order approximation of the considered concentration was calculated by replacement of the considered function on not yet known average value α_{1C} in the right side of the equation (1), i.e. C (z,t)$\to\alpha_{1C}$. The replacement with future integration of the obtain relation on time gives a possibility to obtain the first-order approximation of concentration of the considered salts in the following form

$$C_1(z,t) = C_0 - \alpha_{1C}\int_0^t\left[K(z,\tau) + div\left(\vec{V}\right)\right] d\,\tau \,. (11)$$

The second-order approximation of the considered concentration of salts was determine by standard replacing of the required function in the right side of equation (1) on sum of average value of the considered approximation and approximation with the previous order, i.e. C (z,t)$\to\alpha_{2C}$+C_1(z,t). The replacement with future integration on time of the obtained relation gives a possibility to obtain ratio to determine the second-order approximation of the required concentration in the following form

$$C_2(z,t) = C_0 + div\left\{\int_0^t D(T)\, grad\left[C_1(z,\tau)\right] d\,\tau\right\} - \int_0^t K(z,\tau)\left[\alpha_{2C} + C_1(z,\tau)\right] d\,\tau -$$

$$- div\int_0^t \vec{V}\cdot\left[\alpha_{2C} + C_1(z,t)\right] d\,\tau \ldots\ldots (12)$$

In this paper we obtained the required functions as the second-order approximations in the framework of the standard procedure of the method of averaging of function corrections. Usually the second-order approximation is enough good approximations to make qualitative analysis and to obtain quantitative results. All analytical results have been checked by numerical simulation.

III. DISCUSSION

In this section we analyzed transport of the considered an aqueous solution of salts in a xylem on several parameters. Fig. 1 shows dependence of concentration of the considered salts on it's coefficient of diffusion. Increasing of numbers of curves corresponds to increasing of pressure in the considered system. The figure shows monotonous decreasing of the considered concentration. Increasing of value of diffusion coefficient leads to increasing of velocity of transport of salts with decreasing of their values at fixed value of their source. Fig. 2 shows dependence of concentration of salts on kinematics viscosity ν. Decreasing of the concentration with increasing of the viscosity was obtained due to deceleration of the transport of salts to the considered area. Fig. 3 shows dependence of concentration of salts on it's velocity in inlet of xylem V_0. Increasing of number of curve corresponds to decreasing of pressure. Increasing of velocity V_0 leads to increasing of concentration of salts of xylem at fixed values of other parameters.

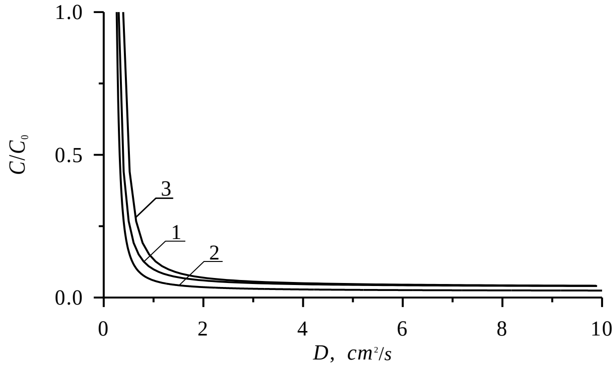

Fig. 1: Dependence of Concentration of the Considered Salts on it is Coefficient of Diffusion D

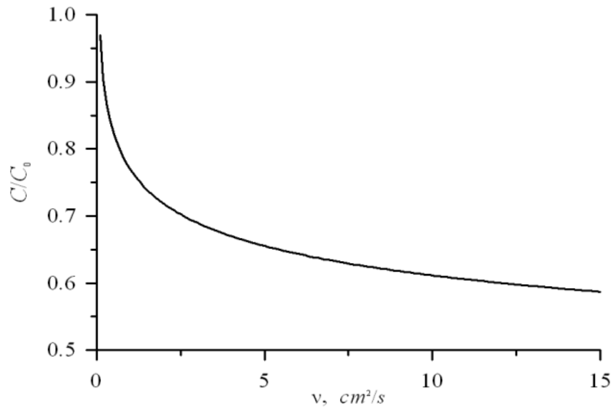

Fig. 2: Dependence of Concentration of Salts on Kinematics Viscosity ν

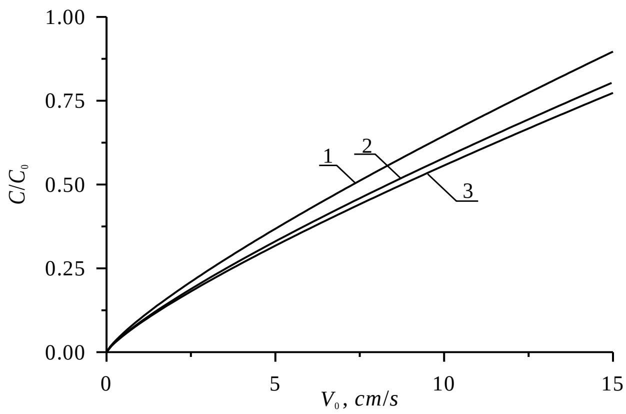

Fig. 3: Dependence of Concentration of Salts on its Velocity in Inlet of Xylem V0

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IV. CONCLUSION

In this paper we consider a model for the vertical lifting of an aqueous solution of salts in a xylem. Based on the model we analyzed changes of movement of the considered solution depending on various parameters. In this paper we also consider an analytical approach of analysis of the considered model. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered processes.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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REFERENCES

1. I.A. Kipnis, Yu.M. Vernigorov, Mathematical modeling of the lifting of water in capillaries shaped like a cylindrical vertical spiral, Bulletin of the far eastern state technical university. 14 (2) (2014) 139-144. https://doi.org/10.12737/4542
2. I.A. Kipnis, Yu.M. Vernigorov, The mechanism of water movement in plant xylem capillaries, Bulletin of the far eastern state technical university. 14 (3) (2014) 77-87. https://doi.org/10.12737/5698
3. B.N. Ugolev, Wood science and forest commodity science (Moscow: Academy, 2010).
4. L.I. Lotova. Botany. Morphology and anatomy of higher plants (Moscow: LIBRICOM, 2010).
5. M. Niu, Zh. Bie, Y. Huang, Root-borne signals and their control of guard cell operation under saline conditions: The role of root signals in stomata regulation, Advances in botanical research, 103, (2022), 195-220. https://doi.org/10.1016/bs.abr.2022.02.014
6. E.L. Pankratov, E.A. Bulaeva, on prognosis of epitaxy from gas phase process for improvement of properties of epitaxial layers, 3D research. 6 (4) (2015) 46-56. https://doi.org/10.1007/s13319-015-0073-4
7. R.A. Talalaev, E.V. Yakovleva, S.Yu. Karpova, Yu.N. Makarov, On low temperature kinetic effects in metal-organic vapor phase epitaxy of III-V compounds, Journal of the crystal growth. 230 (2001) 232-238. https://doi.org/10.1016/S0022-0248(01)01354-9
8. Yu.D. Sokolov, About the definition of dynamic forces in the mine lifting. Applied mechanics.**Vol. 1** (1). P. 23-35 (1955).
9. E.L. Pankratov, Decreasing of depth of p-n-junction in a semiconductor heterostructure by serial radiation processing and microwave annealing, Journal of computational and theoretical nanoscience, 9 (1) (2012) 41-49. https://doi.org/10.1166/jctn.2012.1994
10. Kumar, A., avtar, Dr. R., & Seth, Dr. D. (2019). Mathematical Modeling of Oxygen Transport in Retinal Layers. In International Journal of Emerging Science and Engineering (Vol. 6, Issue 6, pp. 1–4). https://doi.org/10.35940/ijese.f2304.106619
11. Anusha, G., Rao, S. V. P., & Krishna, C. B. R. (2019). Designing Of Modeling and Applications in Typical Engineering Process. In International Journal of Recent Technology and Engineering (IJRTE) (Vol. 8, Issue 2, pp. 2289–2291). . https://doi.org/10.35940/ijrte.b2665.078219
12. Sivaraman, Dr. R., López-Bonilla, Prof. J., & Vidal-Beltrán, S. (2023). On the Polynomial Structure of $rk(n)$. In Indian Journal of Advanced Mathematics (Vol. 3, Issue 2, pp. 4–6). https://doi.org/10.54105/ijam.a1162.103223

AUTHOR PROFILE

Evgeny Leonidovich Pankratov, was educated Nizhny Novgorod state university (Nizhny Novgorod city, Russia) with full doctor degree in physics and mathematics. Now he has a position of a full professor. Area of scientific interests of Evgeny Leonidovich Pankratov is prognosis of processes in physics, biology and economics with appropriate development of models and analytical approaches for solution of equations, which were used in the considered models. Now he have 580 published papers in area of his research.