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A Proof for Fermat's Last Theorem using an Auxiliary Fermat's Equation

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Abstract: ***Fermat's Last Theorem states that there exists no three positive integers x, y and z satisfying the equation $x^n + y^n = z^n$, where n is any integer > 2. Fermat and Euler had already proved the theorem for the exponents n = 4 and n = 3 in the equations $x^4 + y^4 = z^4$ and $x^3 + y^3 = z^3$ respectively. Hence taking into account of the same, it is enough to prove the theorem for the exponent n = p, where p is any prime > 3. In this proof, we have hypothesized that r, s and t are positive integers in the equation $r^p + s^p = t^p$ where p is any prime >3 and prove the theorem by the method of contradiction. To support the proof in the above equation we have used an Auxiliary equation $x^3 + y^3 = z^3$. The two equations are linked by means of transformation equations. Solving the transformation equations we prove the theorem.***

Keywords: Transformation Equations.

Mathematics Subject Classification 2010: 11A–XX.

I. INTRODUCTION

During 1637, the French mathematician Pierre de Fermat conjectured in the margin of a book that the equation $x^n + y^n = z^n$ has no integral solutions for x, y and z, where $n > 2$. He mentioned therein that he himself had found a marvelous solution to his conjecture, but the margin was too narrow to contain it. However his proof is available only for $x^4 + y^4 = z^4$ using infinite descent method. Subsequently Euler in 1770 proved the theorem for $x^3 + y^3 = z^3$ [1]. Later on Sophie Germain around 1819 proved the theorem for a general case, and subsequently E.E. Kummer proved the theorem for regular primes [2]. Number theory developed leaps and bounds in search of a proof for the theorem [3]. During the last 350 years many eminent mathematicians around the world had contributed to the theorem Finally Professor Andrew Wiles, proved the theorem completely in 1995 and his proof was published in the Journal 'Annals of Mathematics'. His proof involved highly advanced mathematical tools and techniques [4].

II. ASSUMPTIONS

1) We initially hypothesize that all r, s and t are non-zero integers satisfying the equation
$$r^p + s^p = t^p$$
where p is any prime > 3, and establish a contradiction in this proof. We can have $\gcd(r, s, t) = 1$
2) We have used the Auxiliary equation $x^3 + y^3 = z^3$ for supporting the proof in the main equation $r^p + s^p = t^p$. Since we are proving the theorem only in the main equation, we have the choice of assigning suitable numerical values for x, y and z^3. Without loss of generality we can have x and y as positive integers; z^3 a positive integer; both z and z^2 irrational. In this proof we have assigned the values as $x = 11$; $y = 53$; $z^3 = 11^3 + 53^3 = 8^2 \times 2347$. We have created transformation equations to the above two equations and linked them through parameters called a, b, c, d, e and f.
3) We have incorporated the Ramanujan-Nagell equation $2^n = 7 + \ell^2$, into the transformation equations where we consider n is odd and $\ell > 1$.
4) In the transformation equations we have used E and R as any distinct odd primes, both coprime to each of x, y, z^3, $r.\ s, t$, 7 and ℓ; and $F^{1/3} = \left(2^{3n/2} rst\right)$.

Proof. By random trials, we have created the following equations,

$$\left(\frac{a\sqrt{s} + b\sqrt{\ell^{5/3}}}{\sqrt{53}}\right)^2 + \left(\frac{c\sqrt{7^{1/3}t^p} + d\sqrt{E^{5/3}}}{\sqrt{2347}}\right)^2 = \left(\frac{e\sqrt{F^{1/3}} + f\sqrt{E^{1/3}}}{\sqrt{\ell^{7/3}}}\right)^2$$

and

$$\left(a\sqrt{2^{n/2}s^p} - b\sqrt{F^{1/3}}\right)^2 + \left(\frac{c\sqrt{t} - d\sqrt{R^{1/3}}}{\sqrt{2^{3n/2}}}\right)^2 = \left(\frac{e\sqrt{r^p} - \sqrt{11R^{5/3}}}{\sqrt{7^{5/3}}}\right)^2 \quad (1)$$

as the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively through the parameters called a, b, c, d, e and f. We have used the Ramanujan-Nagell equation $2^n = 7 + \ell^2$ and for this proof we are using the solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$. E and R are any distinct odd primes, each coprime to each of x, y, z^3, r, s, t, 7 and ℓ and $F^{1/3} = (2^{3n/2}rst)$; $x = 11$; $y = 53$; $z^3 = 11^3 + 53^3 = 8^2 \times 2347$. We can choose x, y, z^3 such that they are coprime to r, s and t; otherwise, we have the choice of assigning alternate values for x, y, z^3 such that $x = 17$; $y = 47$; $z^3 = 17^3 + 47^3 = 8^2 \times 1699$ and so on.

From equation (1), we get

$$a\sqrt{s} + b\sqrt{\ell^{5/3}} = \sqrt{53x^3} \quad (2)$$

$$a\sqrt{2^{n/2}s^p} - b\sqrt{F^{1/3}} = \sqrt{r^p} \quad (3)$$

$$c\sqrt{7^{1/3}t^p} + d\sqrt{E^{5/3}} = \sqrt{2347y^3} \quad (4)$$

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$$c\sqrt{t}-d\sqrt{R^{1/3}}=\sqrt{2^{3n/2}s^{p}} \qquad (5)$$

$$e\sqrt{F^{1/3}}+f\sqrt{E^{1/3}}=\sqrt{\ell^{7/3}z^{3}} \qquad (6)$$

and $$e\sqrt{r^{p}}-f\sqrt{11R^{5/3}}=\sqrt{7^{5/3}t^{p}} \qquad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a=\left(\sqrt{53F^{1/3}x^{3}}+\sqrt{r^{p}\ell^{5/3}}\right)\Big/\left(\sqrt{F^{1/3}}s+\sqrt{2^{n/2}\times\ell^{5/3}s^{p}}\right)$$

$$b=\left(\sqrt{53\times2^{n/2}x^{3}s^{p}}-\sqrt{r^{p}}s\right)\Big/\left(\sqrt{F^{1/3}}s+\sqrt{2^{n/2}\times\ell^{5/3}s^{p}}\right)$$

$$c=\left(\sqrt{2347R^{1/3}y^{3}}+\sqrt{2^{3n/2}E^{5/3}s^{p}}\right)\Big/\left(\sqrt{(7R)^{1/3}t^{p}}+\sqrt{E^{5/3}t}\right)$$

$$d=\left(\sqrt{2347y^{3}t}-\sqrt{2^{3n/2}7^{1/3}s^{p}t^{p}}\right)\Big/\left(\sqrt{(7R)^{1/3}t^{p}}+\sqrt{E^{5/3}t}\right)$$

$$e=\left(\sqrt{11R^{5/3}\ell^{7/3}z^{3}}+\sqrt{E^{1/3}7^{5/3}t^{p}}\right)\Big/\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)$$

and $$f=\left(\sqrt{\ell^{7/3}z^{3}r^{p}}-\sqrt{F^{1/3}7^{5/3}t^{p}}\right)\Big/\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)$$

From (3) & (7), we have

$$\sqrt{r^{p}}\times\sqrt{r^{p}}=\left(a\sqrt{2^{n/2}s^{p}}-b\sqrt{F^{1/3}}\right)\left(\sqrt{7^{5/3}t^{p}}+f\sqrt{11R^{5/3}}\right)\Big/(e)$$

i.e., $$r^{p}=\left\{(a)\sqrt{2^{n/2}\times7^{5/3}s^{p}t^{p}}+(af)\sqrt{11\times2^{n/2}R^{5/3}s^{p}}-(b)\sqrt{F^{1/3}7^{5/3}t^{p}}-(bf)\sqrt{11F^{1/3}R^{5/3}}\right\}\Big/(e)$$

From (2) & (5), we get

$$\sqrt{s^{p}}\times\sqrt{s^{p}}=\left(\frac{\sqrt{r^{p}}+b\sqrt{F^{1/3}}}{\left(a\sqrt{2^{n/2}}\right)}\right)\left(\frac{c\sqrt{t}-d\sqrt{R^{1/3}}}{\sqrt{2^{3n/2}}}\right)$$

i.e., $$s^{p}=\left\{(c)\sqrt{r^{p}t}-(d)\sqrt{R^{1/3}r^{p}}+(bc)\sqrt{F^{1/3}t}-(bd)\sqrt{F^{1/3}R^{1/3}}\right\}\Big/\left(2^{n}a\right)$$

From (4) & (7), we get

$$\sqrt{t^{p}}\times\sqrt{t^{p}}=\left(\frac{\sqrt{2347y^{3}}-d\sqrt{E^{5/3}}}{\left(c\sqrt{7^{1/3}}\right)}\right)\left(\frac{e\sqrt{r^{p}}-\sqrt{11R^{5/3}}}{\sqrt{7^{5/3}}}\right)$$

i.e., $$t^{p}=\left\{(e)\sqrt{2347y^{3}r^{p}}-\sqrt{11\times2347R^{5/3}y^{3}}-(de)\sqrt{E^{5/3}r^{p}}+(d)\sqrt{11E^{5/3}R^{5/3}}\right\}\Big/(7c)$$

Substituting the above equivalent values of r^p, s^p and t^p in the Fermat's equation $t^p = r^p + s^p$ after multiplying both sides by $\left\{\left(7\times2^{n}\right)(ace)\right\}$, we get

$$\left\{2^{n}(ae)\right\}\left\{(e)\sqrt{2347y^{3}r^{p}}-\sqrt{11\times2347R^{5/3}y^{3}}-(de)\sqrt{E^{5/3}r^{p}}+(d)\sqrt{11E^{5/3}R^{5/3}}\right\}$$
$$=\left\{2^{n}\times7ac\right\}\left\{(a)\sqrt{2^{n/2}7^{5/3}s^{p}t^{p}}+(af)\sqrt{11\times2^{n/2}R^{5/3}s^{p}}-(b)\sqrt{F^{1/3}7^{5/3}t^{p}}-(bf)\sqrt{11F^{1/3}R^{5/3}}\right\}$$
$$+\left\{(7\times ce)\right\}\left\{(c)\sqrt{r^{p}t}-(d)\sqrt{R^{1/3}r^{p}}+(bc)\sqrt{F^{1/3}t}-(bd)\sqrt{F^{1/3}R^{1/3}}\right\} \qquad (8)$$

Our aim is to compute all rational terms in equation (8) and equate them on both sides.
To facilitate this, let us multiply both sides of equation by

$$\left\{\left(\sqrt{F^{1/3}}s+\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right)^{2}\left(\sqrt{7^{1/3}R^{1/3}t^{p}}+\sqrt{E^{5/3}t}\right)^{2}\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)^{2}\right\}$$

for freeing from denominators on the parameters a, b, c, d, e and f, as worked out below, term by term.
I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ae^2\}$

$$=\left(2^{n}\sqrt{2347\times y^{3}r^{p}}\right)\left(\sqrt{F^{1/3}}s+\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right)\left\{\left(7^{1/3}R^{1/3}t^{p}\right)+\left(E^{5/3}t\right)+2\sqrt{t^{p+1}}\sqrt{7^{1/3}E^{5/3}R^{1/3}}\right\}$$
$$\times\left(\sqrt{53F^{1/3}x^{3}}+\sqrt{\ell^{5/3}r^{p}}\right)\left\{\left(11R^{5/3}\ell^{7/3}z^{3}\right)+\left(7^{5/3}E^{1/3}t^{p}\right)+2\sqrt{11R^{5/3}\ell^{7/3}z^{3}}\sqrt{7^{5/3}E^{1/3}t^{p}}\right\}$$

On multiplying by

$$\left\{\left(2^{n}\sqrt{2347\times y^{3}r^{p}}\right)\sqrt{2^{n/2}\ell^{5/3}s^{p}}\left(2\sqrt{t^{p+1}}\sqrt{7^{1/3}E^{5/3}R^{1/3}}\right)\sqrt{53F^{1/3}x^{3}}\left(2\sqrt{11R^{5/3}\ell^{7/3}z^{3}}\sqrt{7^{5/3}E^{1/3}t^{p}}\right)\right\}$$

we get

$$\left\{\left(2^{n+2}\times7ER\ell^{2}t^{p}\right)\sqrt{11x^{3}}\sqrt{53y^{3}}\sqrt{2347z^{3}}\sqrt{2^{n/2}\times F^{1/3}r^{p}s^{p}t}\right\}$$

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which will be rational since $x = 11$; $y = 53$; $z^3 = 8^2 \times 2347$ and $F^{1/3} = (2^{3n/2}\, rst)$.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ae\}$

$$=\left(-2^n\sqrt{11\times 2347\times R^{5/3}y^3}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}s^p}\right)\left\{\left(7^{1/3}R^{1/3}t^p\right)+\left(E^{5/3}t\right)+2\sqrt{t^{p+1}}\sqrt{7^{1/3}E^{5/3}R^{1/3}}\right\}$$
$$\times\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^p}\right)\left(\sqrt{53F^{1/3}x^3}+\sqrt{\ell^{5/3}r^p}\right)\left(\sqrt{11R^{5/3}\ell^{7/3}z^3}+\sqrt{7^{5/3}E^{1/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-2^n\sqrt{11\times 2347\times R^{5/3}y^3}\right)\sqrt{2^{n/2}\ell^{5/3}s^p}\left(2\sqrt{t^{p+1}}\sqrt{7^{1/3}E^{5/3}R^{1/3}}\right)\sqrt{E^{1/3}r^p}\sqrt{53F^{1/3}x^3}\sqrt{7^{5/3}E^{1/3}t^p}\right\}$$

we get

$$\left\{-\left(2^{n+1}\times 7ERt^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times 2^{n/2}\ell^{5/3}\left(FE\right)^{1/3}r^ps^pt}\right\}$$

which is irrational.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ade^2\}$

$$=\left(-2^n\sqrt{E^{5/3}r^p}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}s^p}\right)\left(\sqrt{7^{1/3}R^{1/3}t^p}+\sqrt{E^{5/3}t}\right)\left(\sqrt{53F^{1/3}x^3}+\sqrt{\ell^{5/3}r^p}\right)$$
$$\times\left(\sqrt{2347y^3t}-\sqrt{2^{3n/2}7^{5/3}s^pt^p}\right)\left\{\left(11R^{5/3}\ell^{7/3}z^3\right)+\left(7^{5/3}E^{1/3}t^p\right)+2\sqrt{11R^{5/3}\ell^{7/3}z^3}\sqrt{7^{5/3}E^{1/3}t^p}\right\}$$

On multiplying by

$$\left\{\left(-2^n\sqrt{E^{5/3}r^p}\right)\sqrt{2^{n/2}\ell^{5/3}s^p}\sqrt{7^{1/3}R^{1/3}t^p}\sqrt{53F^{1/3}x^3}\sqrt{2347y^3t}\left(2\sqrt{11R^{5/3}\ell^{7/3}z^3}\sqrt{7^{5/3}E^{1/3}t^p}\right)\right\}$$

we get

$$\left\{-\left(2^{n+1}\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}$$

Which will be rational since $x = 11$; $y = 53$; $z^3 = 8^2 \times 2347$ and $F^{1/3} = (2^{3n/2}\, rst)$.

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ade\}$

$$=\left(2^n\sqrt{11\times E^{5/3}R^{5/3}}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}s^p}\right)\left(\sqrt{7^{1/3}R^{1/3}t^p}+\sqrt{E^{5/3}t}\right)\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^p}\right)$$
$$\times\left(\sqrt{53F^{1/3}x^3}+\sqrt{\ell^{5/3}r^p}\right)\left(\sqrt{2347y^3t}-\sqrt{2^{3n/2}7^{1/3}s^pt^p}\right)\left(\sqrt{11R^{5/3}\ell^{7/3}z^3}+\sqrt{7^{5/3}E^{1/3}t^p}\right)$$

On multiplying by

$$\left\{\left(2^n\sqrt{11\times E^{5/3}R^{5/3}}\right)\sqrt{2^{n/2}\ell^{5/3}s^p}\sqrt{7^{1/3}R^{1/3}t^p}\sqrt{E^{1/3}r^p}\sqrt{53F^{1/3}x^3}\sqrt{2347y^3t}\sqrt{7^{5/3}E^{1/3}t^p}\right\}$$

we get

$$\left\{\left(2^n\times 7ERt^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times 2^{n/2}\times\left(FE\right)^{1/3}\ell^{5/3}r^ps^pt}\right\}$$

which is irrational since $F^{1/3} = (2^{3n/2}\, rst)$.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2c\}$

$$=\left(7\times 2^n\right)\sqrt{2^{n/2}\times 7^{5/3}s^pt^p}\left(\sqrt{7^{1/3}R^{1/3}t^p}+\sqrt{E^{5/3}t}\right)\left\{\left(11F^{1/3}R^{5/3}\right)+\left(E^{1/3}s^p\right)+2\sqrt{11F^{1/3}R^{5/3}}\sqrt{E^{1/3}r^p}\right\}$$
$$\times\left\{\left(53F^{1/3}x^3\right)+\left(\ell^{5/3}r^p\right)+2\sqrt{53F^{1/3}x^3}\sqrt{\ell^{5/3}r^p}\right\}\left(\sqrt{2347R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(7\times 2^n\right)\sqrt{2^{n/2}\times 7^{5/3}s^pt^p}\sqrt{7^{1/3}R^{1/3}t^p}\left(2\sqrt{11F^{1/3}R^{5/3}}\sqrt{E^{1/3}r^p}\right)\left(53F^{1/3}x^3\right)\sqrt{2^{3n/2}E^{5/3}s^p}\right\}$$

we get

$$\left\{\left(2^{2n+1}\times 7^2\times 53ERx^3s^pt^p\right)\sqrt{11Fr^p}\right\}\qquad\left(\because\; F^{1/3}\sqrt{F^{1/3}}=\sqrt{F}\right)$$

which is irrational.

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(a^2cf\}$

$$=\left(7\times 2^n\right)\sqrt{11\times 2^{n/2}\times R^{5/3}s^p}\left(\sqrt{7^{1/3}R^{1/3}t^p}+\sqrt{E^{5/3}t}\right)\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^p}\right)\left(\sqrt{\ell^{7/3}z^3r^p}-\sqrt{F^{1/3}7^{5/3}t^p}\right)$$
$$\times\left\{\left(53F^{1/3}x^3\right)+\left(\ell^{5/3}r^p\right)+2\sqrt{53F^{1/3}x^3}\sqrt{\ell^{5/3}r^p}\right\}\left(\sqrt{2347R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(7\times 2^n\right)\sqrt{11\times 2^{n/2}\times R^{5/3}s^p}\sqrt{E^{5/3}t}\sqrt{E^{1/3}r^p}\left(2\sqrt{53F^{1/3}x^3}\sqrt{\ell^{5/3}r^p}\right)\sqrt{2347R^{1/3}y^3}\sqrt{\ell^{7/3}z^3r^p}\right\}$$

we get

$$\left\{2^{n+1}\times 7ERr^{p}\ell^{2}\sqrt{11x^{3}}\sqrt{53y^{3}}\sqrt{2347\times z^{3}}\sqrt{2^{n/2}F^{1/3}r^{p}s^{p}t}\right\}$$

which is rational.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)c\}$

$$=\left(-7\times 2^{n}\right)\sqrt{F^{1/3}\times 7^{5/3}t^{p}}\left(\sqrt{7^{1/3}R^{1/3}t^{p}}+\sqrt{E^{5/3}t}\right)\left\{\left(11F^{1/3}R^{5/3}\right)+\left(E^{1/3}r^{p}\right)+2\sqrt{11F^{1/3}R^{5/3}}\sqrt{E^{1/3}r^{p}}\right\}$$
$$\times\left(\sqrt{53F^{1/3}x^{3}}+\sqrt{\ell^{5/3}r^{p}}\right)\left(\sqrt{53\times 2^{n/2}x^{3}s^{p}}-\sqrt{r^{p}s}\right)\left(\sqrt{2347R^{1/3}y^{3}}+\sqrt{2^{3n/2}E^{5/3}s^{p}}\right)$$

On multiplying by

$$\left\{\left(-7\times 2^{n}\right)\sqrt{F^{1/3}\times 7^{5/3}t^{p}}\sqrt{7^{1/3}R^{1/3}t^{p}}\left(2\sqrt{11F^{1/3}R^{5/3}}\sqrt{E^{1/3}r^{p}}\right)\sqrt{53F^{1/3}x^{3}}\sqrt{53\times 2^{n/2}x^{3}s^{p}}\sqrt{2^{3n/2}E^{5/3}s^{p}}\right\}$$

we get

$$\left\{-\left(2^{2n+1}\times 53\times 7^{2}ERx^{3}s^{p}t^{p}\right)\sqrt{11\times Fr^{p}}\right\}$$

This term gets cancelled with that term worked out under I term in RHS of equation (8).

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)cf\}$

$$=\left(-7\times 2^{n}\right)\sqrt{11F^{1/3}R^{5/3}}\left(\sqrt{7^{1/3}R^{1/3}t^{p}}+\sqrt{E^{5/3}t}\right)\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)\left(\sqrt{\ell^{7/3}z^{3}r^{p}}-\sqrt{F^{1/3}7^{5/3}t^{p}}\right)$$
$$\times\left(\sqrt{53F^{1/3}x^{3}}+\sqrt{\ell^{5/3}r^{p}}\right)\left(\sqrt{53\times 2^{n/2}x^{3}s^{p}}-\sqrt{r^{p}s}\right)\left(\sqrt{2347R^{1/3}y^{3}}+\sqrt{2^{3n/2}E^{5/3}s^{p}}\right)$$

On multiplying by

$$\left\{\left(-7\times 2^{n}\right)\sqrt{11F^{1/3}R^{5/3}}\sqrt{E^{5/3}t}\sqrt{E^{1/3}r^{p}}\sqrt{\ell^{5/3}r^{p}}\sqrt{53\times 2^{n/2}x^{3}s^{p}}\sqrt{2347R^{1/3}y^{3}}\sqrt{\ell^{7/3}z^{3}r^{p}}\right\}$$

we get

$$\left\{-\left(2^{n}\times 7ERr^{p}\ell^{2}\right)\sqrt{11x^{3}}\sqrt{53y^{3}}\sqrt{2347\times z^{3}}\sqrt{2^{n/2}F^{1/3}r^{p}s^{p}t}\right\}$$

which is rational.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{c^{2}e\}$

$$=\left(7\sqrt{r^{p}t}\right)\left\{\left(F^{1/3}s\right)+\left(\ell^{5/3}s^{p}\sqrt{2^{n}}\right)+2\sqrt{F^{1/3}s}\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right\}\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)$$
$$\times\left\{\left(2347\times R^{1/3}y^{3}\right)+\left(E^{5/3}s^{p}\sqrt{2^{3n}}\right)+2\sqrt{2347\times R^{1/3}y^{3}}\sqrt{2^{3n/2}E^{5/3}s^{p}}\right\}\left(\sqrt{11R^{5/3}\ell^{7/3}z^{3}}+\sqrt{7^{5/3}E^{1/3}t^{p}}\right)$$

On multiplying by

$$\left\{\left(7\sqrt{r^{p}t}\right)\left(2\sqrt{F^{1/3}s}\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right)\sqrt{11F^{1/3}R^{5/3}}\left(2347\times R^{1/3}y^{3}\right)\sqrt{11R^{5/3}\ell^{7/3}z^{3}}\right\}$$

we get

$$\left\{\left(2\times 7\times 11\times 2347R^{2}y^{3}\ell^{2}\right)\sqrt{s^{p+1}}\sqrt{2^{n/2}F^{2/3}r^{p}tz^{3}}\right\}$$

which is irrational.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(cd)e\}$ is

$$=\left(-7\sqrt{R^{1/3}r^{p}}\right)\left\{\left(F^{1/3}s\right)+\left(\ell^{5/3}s^{p}\sqrt{2^{n}}\right)+2\sqrt{F^{1/3}s}\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right\}\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)$$
$$\times\left(\sqrt{2347\times R^{1/3}y^{3}}+\sqrt{2^{3n/2}E^{5/3}s^{p}}\right)\left(\sqrt{2347y^{3}t}-\sqrt{2^{3n/2}7^{1/3}s^{p}t^{p}}\right)\left(\sqrt{11R^{5/3}\ell^{7/3}z^{3}}+\sqrt{7^{5/3}E^{1/3}t^{p}}\right)$$

On multiplying by

$$\left\{\left(-7\sqrt{R^{1/3}r^{p}}\right)\left(F^{1/3}s\right)\sqrt{11F^{1/3}R^{5/3}}\sqrt{2^{3n/2}E^{5/3}s^{p}}\left(-\sqrt{2^{3n/2}7^{1/3}s^{p}t^{p}}\right)\sqrt{7^{5/3}E^{1/3}t^{p}}\right\}$$

we get

$$\left\{\left(2^{n}\times 7^{2}ERs^{p+1}t^{p}\right)\sqrt{11\times 2^{n}Fr^{p}}\right\}\qquad\left(\because\; F^{1/3}\sqrt{F^{1/3}}=\sqrt{F}\right)$$

which is irrational.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bc^{2}e)\}$ is

$$=\left(7\sqrt{F^{1/3}t}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}s^{p}}\right)\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^{p}}\right)\left(\sqrt{53\times 2^{n/2}x^{3}s^{p}}-\sqrt{r^{p}s}\right)$$

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$$\times\left\{\left(2347\times R^{1/3}y^3\right)+\left(E^{5/3}s^p\sqrt{2^{3n}}\right)+2\sqrt{2347\times R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right\}\left(\sqrt{11R^{5/3}\ell^{7/3}z^3}+\sqrt{7^{5/3}E^{1/3}t^p}\right)$$

On multiplying by

$$\left\{\left(7\sqrt{F^{1/3}t}\right)\sqrt{2^{n/2}\ell^{5/3}s^p}\sqrt{E^{1/3}r^p}\sqrt{53\times 2^{n/2}x^3s^p}\left(2\sqrt{2347\times R^{1/3}y^3}\sqrt{2^{3n/2}E^{5/3}s^p}\right)\sqrt{11R^{5/3}\ell^{7/3}z^3}\right\}$$

we get

$$\left\{\left(2^{n+1}\times 7\ell^2s^p\right)ER\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}$$

which is rational.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)e\}$ is

$$=\left(-7\sqrt{F^{1/3}R^{1/3}}\right)\left(\sqrt{F^{1/3}s}+\sqrt{2^{n/2}\ell^{5/3}s^p}\right)\left(\sqrt{11F^{1/3}R^{5/3}}+\sqrt{E^{1/3}r^p}\right)\left(\sqrt{53\times 2^{n/2}x^3s^p}-\sqrt{r^ps}\right)$$

$$\times\left(\sqrt{2347\times R^{1/3}y^3}+\sqrt{2^{3n/2}E^{5/3}s^p}\right)\left(\sqrt{2347y^3t}-\sqrt{2^{3n/2}7^{1/3}s^pt^p}\right)\left(\sqrt{11R^{5/3}\ell^{7/3}z^3}+\sqrt{7^{5/3}E^{1/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-7\sqrt{F^{1/3}R^{1/3}}\right)\sqrt{2^{n/2}\ell^{5/3}s^p}\sqrt{E^{1/3}r^p}\sqrt{53\times 2^{n/2}x^3s^p}\sqrt{2^{3n/2}E^{5/3}s^p}\sqrt{2347y^3t}\sqrt{11R^{5/3}\ell^{7/3}z^3}\right\}$$

we get

$$\left\{-\left(2^n\times 7\ell^2s^p\right)ER\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}$$

which will be rational.

Sum of all rational part in LHS of equation (8)

$$=\left\{\left(2^{n+2}\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347z^3}\sqrt{2^{n/2}\times F^{1/3}r^ps^pt}\right\}\qquad\text{(vide I term)}$$

$$-\left\{\left(2^{n+1}\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}\qquad\text{(vide III term)}$$

$$=\left\{\left(2^{n+1}\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}$$

Sum of all rational part in RHS of equation (8)

$$=\left\{\left(2^n\times 7ER\ell^2\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\left(r^p+s^p\right)\right\}$$

(combining II, IV, VII and VIII terms)

Equating the rational terms on both sides of equation (8), we get

$$\left\{\left(2^{n+1}\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}$$

$$=\left\{\left(2^n\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}\qquad\left(\because\quad r^p+s^p=t^p\right)$$

That is

$$\left\{\left(2^n\times 7ER\ell^2t^p\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right\}=0$$

Dividing both sides by

$$\left\{\left(2^n\times 7ER\ell^2\right)\sqrt{11x^3}\sqrt{53y^3}\sqrt{2347\times z^3}\right\}$$

we get

$$\left(t^p\sqrt{2^{n/2}F^{1/3}r^ps^pt}\right)=0\quad\left(\because\quad 2^n\times t^{p+1}\sqrt{(rs)^{p+1}}=0\right)$$

$$\left(\because\quad F^{1/3}=2^{3n/2}rst\right)$$

That is, either $r = 0$; or $s = 0$; or $t = 0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$, where p is any prime > 3, thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

Equation (8) was derived from the transformation equations by substituting the equivalent values of r^p, S^p & t^p in the Fermat equation r^p

$+ s^p = t^p$.

The only main hypothesis that we made in this proof, namely, r, s and t are non-zero integers has been shattered by the result $rst = 0$, thus proving the theorem.

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