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In Search of an Elementary Proof for Fermat's Last Theorem

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***Abstract:** Fermat's Last Theorem states that the equation $x^n + y^n = z^n$ has no solution for x, y and z as positive integers, where n is any positive integer > 2. Taking the proofs of Fermat and Euler for the exponents n = 4 and n = 3, it would suffice to prove the theorem for the exponent n = p, where p is any prime > 3. We hypothesize that r, s and t are positive integers satisfying the equation $r^p + s^p = t^p$ and establish a contradiction in this proof. We include another Auxiliary equation $x^3 + y^3 = z^3$ and connects these two equations by using the transformation equations. On solving the transformation equation we prove rst = 0, thus proving that only a trivial solution exists in the main equation $r^p + s^p = t^p$.*

***Keywords:** Transformation Equations To Two Fermat's Equations, Mathematics Subject Classification 2010: 11A–XX.*

I. INTRODUCTION

Around 1637, Pierre de Fermat a French Mathematician, wrote in the margin of his book, claiming that he has found a marvelous proof for the equation $x^n + y^n = z^n$, but the margin was too narrow to contain it. His proof is available only for the equation $x^4 + y^4 = z^4$, which he had proved using "infinite descent" method. Later on Euler proved the theorem in the equation $x^3 + y^3 = z^3$ [1].

Many mathematicians like Dirichlet, Legendre, Gabril Lame proved the theorem for the exponents n = 5 and n = 7. Around 1820, Sophie Germain gave a remarkable proof for $x^\ell + y^\ell = z^\ell$ where ℓ and $(2\ell+1)$ are both primes and $\ell \neq xyz$. Ernst Kummer made the first substantial step in proving Fermt's Last theorem for Regular Primes. Many mathematicians worked on this theorem by which number theory developed leaps and bounds. Mathematicians found a close relationship between Fermat's Last theorem and Elliptic curve [2]. Finally in 1995 Andrew Wiles proved the theorem completely [3]. Many mathematicians have analysed and explained the theorem in all aspects [4]. In this proof, we are trying for an alternative elementary proof for Fermat's Last theorem.

II. ASSUMPTIONS

1. We presume that all r, s and t are non-zero positive integers in the equation $r^p + s^p = t^p$ where p is any prime > 3, and establish a contradiction.
2. For supporting the proof in the above equation, we are using another auxiliary equation $x^3 + y^3 = z^3$ (already proved) and we connect the two equations by means of transformation equations by using the parameters called a, b, c, d, e and f.
3. Since we are proving the theorem only in the equation $r^p + s^p = t^p$, we have the choice of assigning numerical values for the equation $x^3 + y^3 = z^3$. In this proof we give the values x = 29; y = 71; $z^3 = 29^3 + 71^3 = 10^2 \times 3823$ for convenience.
4. We have used the Ramanujan-Nagell equation $2^n = 7 + \ell^2$, which has just five solutions given by (n, ℓ) = {(3,1); (4,3); (5,5); (7,11); (15,181)}.
5. In this proof, we have considered n as an odd integer and $\ell > 1$, using only one of the solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$.
6. We have used F, E and R in the transformation equations, which we define as distinct odd primes each coprime to each of x, y, z^3, r. s and t,7 and ℓ.

Proof. By trials, we have created the following equations

$$\left(a\sqrt{2^{3n/2}t^p} + b\sqrt{71F^{1/3}}\right)^2 + \left(c\sqrt{3823} + d\sqrt{R}\right)^2 = \left(e\sqrt{F^{5/3}} + f\sqrt{29}\right)^2$$

and

$$\left(\frac{a\sqrt{R} - b\sqrt{F}}{\sqrt{7^{1/3}}}\right)^2 + \left(\frac{c\sqrt{7^{5/3}r^p} - d\sqrt{E^{5/3}}}{\sqrt{\ell^{5/3}}}\right)^2 = \left(\frac{e\sqrt{\ell^{7/3}s^p} - f\sqrt{E^{1/3}}}{\sqrt{2^{n/2}}}\right)^2 \quad \ldots \quad (1)$$

as the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively through the parameters called a, b, c, d, e and f. Here F, E and R are distinct odd primes, each coprime to each of each x, y, z^3, r, s and ℓ; $2^n = 7 + \ell^2$ (Ramanujan-Nagell equation) taking n as odd and $\ell > 1$ for this proof and we assign x =29; y = 71; $z^3 = 29^3 + 71^3 = 10^2 \times 3829$.

From equation (1), we get

$$a\sqrt{2^{3n/2}t^p} + b\sqrt{71F^{1/3}} = \sqrt{x^3} \quad \ldots \quad (2)$$

$$a\sqrt{R} - b\sqrt{F} = \sqrt{7^{1/3}r^p} \quad \ldots \quad (3)$$

$$c\sqrt{3823} + d\sqrt{R} = \sqrt{y^3} \quad \ldots \quad (4)$$

$$c\sqrt{7^{5/3}r^p} - d\sqrt{E^{5/3}} = \sqrt{\ell^{5/3}s^p} \quad \ldots \quad (5)$$

$$e\sqrt{F^{5/3}} + f\sqrt{29} = \sqrt{z^3} \quad \ldots \quad (6)$$

$$\text{and } e\sqrt{\ell^{7/3}s^p} - f\sqrt{E^{1/3}} = \sqrt{2^{n/2}t^p} \quad \ldots \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

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$$a=\left(\sqrt{Fx^3}+\sqrt{71\times 7^{1/3}F^{1/3}r^p}\right)\Big/\left(\sqrt{2^{3n/2}Ft^p}+\sqrt{71\times F^{1/3}R}\right)$$
$$b=\left(\sqrt{Rx^3}-\sqrt{2^{3n/2}\times 7^{1/3}r^pt^p}\right)\Big/\left(\sqrt{2^{3n/2}Ft^p}+\sqrt{71F^{1/3}R}\right)$$
$$c=\left(\sqrt{E^{5/3}y^3}+\sqrt{R\ell^{5/3}s^p}\right)\Big/\left(\sqrt{3823\times E^{5/3}}+\sqrt{7^{5/3}Rr^p}\right)$$
$$d=\left(\sqrt{7^{5/3}y^3r^p}-\sqrt{3823\ell^{5/3}s^p}\right)\Big/\left(\sqrt{3823E^{5/3}}+\sqrt{7^{5/3}Rr^p}\right)$$
$$e=\left(\sqrt{E^{1/3}z^3}+\sqrt{2^{n/2}29t^p}\right)\Big/\left(\sqrt{F^{5/3}E^{1/3}}+\sqrt{29\ell^{7/3}s^p}\right)$$
$$\text{and }\ f=\left(\sqrt{\ell^{7/3}z^3s^p}-\sqrt{F^{5/3}\times 2^{n/2}t^p}\right)\Big/\left(\sqrt{F^{5/3}E^{1/3}}+\sqrt{29\ell^{7/3}s^p}\right)$$

From (3) & (5), we have

$$\sqrt{r^p}\times\sqrt{r^p}=\left(a\sqrt{R}-b\sqrt{F}\right)\left(\sqrt{\ell^{5/3}s^p}+d\sqrt{E^{5/3}}\right)\Big/(7c)$$
$$\text{i.e., }\quad r^p=\left\{(a)\sqrt{R\ell^{5/3}s^p}+(ad)\sqrt{RE^{5/3}}-(b)\sqrt{F\ell^{5/3}s^p}-(bd)\sqrt{FE^{5/3}}\right\}\Big/(7c)$$

From (5) & (7), we get

$$\sqrt{s^p}\times\sqrt{s^p}=\left(c\sqrt{7^{5/3}r^p}-d\sqrt{E^{5/3}}\right)\left(\sqrt{2^{n/2}t^p}+f\sqrt{E^{1/3}}\right)\Big/\left(e\ell^2\right)$$
$$\text{i.e., }\quad s^p=\left\{(c)\sqrt{7^{5/3}2^{n/2}r^pt^p}+(cf)\sqrt{7^{5/3}E^{1/3}r^p}-(d)\sqrt{2^{n/2}E^{5/3}t^p}-(df)E\right\}\Big/\left(e\ell^2\right)$$

From (2) & (7), we get

$$\sqrt{t^p}\times\sqrt{t^p}=\left(\sqrt{x^3}-b\sqrt{71F^{1/3}}\right)\left(e\sqrt{\ell^{7/3}s^p}-f\sqrt{E^{1/3}}\right)\Big/\left(2^n\times a\right)$$
$$\text{i.e., }\quad t^p=\left\{(e)\sqrt{x^3\ell^{7/3}s^p}-(f)\sqrt{E^{1/3}x^3}-(be)\sqrt{71\times F^{1/3}\ell^{7/3}s^p}+(bf)\sqrt{71\times F^{1/3}E^{1/3}}\right\}\Big/\left(2^n\times a\right)$$

Substituting the above new values of r^p, s^p and t^p in the Fermat's equation $r^p + s^p = t^p$ after multiplying both sides by $\left\{\left(7\times 2^n\ell^2\right)(ace)\right\}$, we get

$$\left\{\left(2^n\ell^2\right)(ae)\right\}\left\{(a)\sqrt{R\ell^{5/3}s^p}+(ad)\sqrt{R\times E^{5/3}}-(b)\sqrt{F\ell^{5/3}s^p}-(bd)\sqrt{FE^{5/3}}\right\}$$
$$+\left\{\left(7\times 2^n\right)(ac)\right\}\left\{(c)\sqrt{7^{5/3}\times 2^{n/2}r^pt^p}+(cf)\sqrt{7^{5/3}E^{1/3}r^p}-(d)\sqrt{2^{n/2}E^{5/3}t^p}-(df)E\right\}$$
$$=\left\{\left(7\ell^2\right)(ce)\right\}\left\{(e)\sqrt{x^3\ell^{7/3}s^p}-(f)\sqrt{E^{1/3}x^3}-(be)\sqrt{71\times F^{1/3}\ell^{7/3}s^p}+(bf)\sqrt{71F^{1/3}E^{1/3}}\right\}\quad\dots\quad(8)$$

Our aim is to compute all rational terms in equation (8) and equate them on both sides.

To facilitate this, let us multiply both sides of equation (8) by

$$\left\{\left(\sqrt{2^{3n/2}Ft^p}+\sqrt{71\times F^{1/3}R}\right)^2\left(\sqrt{3823\times E^{5/3}}+\sqrt{7^{5/3}Rr^p}\right)^2\left(\sqrt{F^{5/3}E^{1/3}}+\sqrt{29\ell^{7/3}s^p}\right)^2\right\}$$

for freeing from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* , and again we multiply both sides by $\left(F^{1/3}\sqrt{y}\right)$ for getting some rational terms.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2e\}$

$$=\left(2^n\ell^2\right)\sqrt{R\ell^{5/3}s^p}\left(\sqrt{E^{1/3}\times F^{5/3}}+\sqrt{29\ell^{7/3}s^p}\right)\left\{\left(3823\times E^{5/3}\right)+\left(7^{5/3}Rr^p\right)+2\sqrt{3823\times E^{5/3}}\sqrt{7^{5/3}Rr^p}\right\}$$
$$\times\left(F^{1/3}\sqrt{y}\right)\left\{\left(Fx^3\right)+\left(71\times 7^{1/3}F^{1/3}r^p\right)+2F^{2/3}\sqrt{71\times 7^{1/3}x^3r^p}\right\}\left(\sqrt{E^{1/3}z^3}+\sqrt{29\times 2^{n/2}t^p}\right)$$

On multiplying by

$$\left\{\left(2^n\ell^2\right)\sqrt{R\ell^{5/3}s^p}\sqrt{29\ell^{7/3}s^p}\left(2\sqrt{3823\times E^{5/3}}\sqrt{7^{5/3}Rr^p}\right)\left(2F^{2/3}\sqrt{71\times 7^{1/3}x^3r^p}\right)\left(F^{1/3}\sqrt{y}\right)\sqrt{E^{1/3}z^3}\right\}$$

we get

$$\left\{\left(2^{(n+2)}\times 7FER\ell^4r^ps^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}$$

which is rational ($\because$ $x = 29$; $y = 71$; $z^3 = 10^2 \times 3823$.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2de\}$

$$=\left(2^n\ell^2\right)\sqrt{RE^{5/3}}\left(\sqrt{E^{1/3}\times F^{5/3}}+\sqrt{29\ell^{7/3}s^p}\right)\left(\sqrt{3823\times E^{5/3}}+\sqrt{7^{5/3}Rr^p}\right)\left(F^{1/3}\sqrt{y}\right)$$

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$$\times\left\{\left(Fx^3\right)+\left(71\times7^{1/3}F^{1/3}r^p\right)+2F^{2/3}\sqrt{71\times7^{1/3}x^3r^p}\right\}\left(\sqrt{7^{5/3}y^3r^p}-\sqrt{3823\times\ell^{5/3}s^p}\right)\left(\sqrt{E^{1/3}z^3}+\sqrt{29\times2^{n/2}t^p}\right)$$

On multiplying by

$$\left\{\left(2^n\ell^2\right)\sqrt{RE^{5/3}}\sqrt{29\ell^{7/3}s^p}\sqrt{7^{5/3}Rr^p}\left(F^{1/3}\sqrt{y}\right)\left(2F^{2/3}\sqrt{71\times7^{1/3}x^3r^p}\right)\left(-\sqrt{3823\times\ell^{5/3}s^p}\right)\sqrt{E^{1/3}z^3}\right\}$$

we get

$$\left\{-\left(2^{(n+1)}\times7FER\ell^4r^ps^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}$$

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)e\}$

$$=\left(-2^n\ell^2\right)\sqrt{F\ell^{5/3}s^p}\left(\sqrt{E^{1/3}\times F^{5/3}}+\sqrt{29\ell^{7/3}s^p}\right)\left\{\left(3823\times E^{5/3}\right)+\left(7^{5/3}Rr^p\right)+2\sqrt{3823\times E^{5/3}}\sqrt{7^{5/3}Rr^p}\right\}$$

$$\times\left(\sqrt{Rx^3}-\sqrt{2^{3n/2}7^{1/3}r^pt^p}\right)\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{Fx^3}+\sqrt{71\times7^{1/3}F^{1/3}r^p}\right)\left(\sqrt{E^{1/3}z^3}+\sqrt{29\times2^{n/2}t^p}\right)$$

On multiplying by

$$\left\{\left(-2^n\ell^2\right)\sqrt{F\ell^{5/3}s^p}\sqrt{29\ell^{7/3}s^p}\left(2\sqrt{3823\times E^{5/3}}\sqrt{7^{5/3}Rr^p}\right)\sqrt{Rx^3}\left(F^{1/3}\sqrt{y}\right)\sqrt{71\times7^{1/3}F^{1/3}r^p}\sqrt{E^{1/3}z^3}\right\}$$

we get

$$\left\{-\left(2^{(n+1)}\times7FER\ell^4r^ps^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}$$

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)de)\}$

$$=\left(-2^n\ell^2\right)\sqrt{FE^{5/3}}\left(\sqrt{E^{1/3}\times F^{5/3}}+\sqrt{29\ell^{7/3}s^p}\right)\left(\sqrt{3823\times E^{5/3}}+\sqrt{7^{5/3}Rr^p}\right)\left(F^{1/3}\sqrt{y}\right)$$

$$\times\left(\sqrt{Rx^3}-\sqrt{2^{3n/2}7^{1/3}r^pt^p}\right)\left(\sqrt{Fx^3}+\sqrt{71\times7^{1/3}F^{1/3}r^p}\right)\left(\sqrt{7^{5/3}y^3r^p}-\sqrt{3823\times\ell^{5/3}s^p}\right)\left(\sqrt{E^{1/3}z^3}+\sqrt{29\times2^{n/2}t^p}\right)$$

On multiplying by

$$\left\{\left(-2^n\ell^2\right)\sqrt{FE^{5/3}}\sqrt{29\ell^{7/3}s^p}\sqrt{7^{5/3}Rr^p}\left(F^{1/3}\sqrt{y}\right)\sqrt{Rx^3}\sqrt{71\times7^{1/3}F^{1/3}r^p}\left(-\sqrt{3823\times\ell^{5/3}s^p}\right)\sqrt{E^{1/3}z^3}\right\}$$

we get the rational term given by

$$\left\{\left(2^n\times7FER\ell^4r^ps^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}$$

which is rational.

V term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ac^2\}$

$$=\left(7\times2^n\right)\sqrt{2^{n/2}7^{5/3}r^pt^p}\left(\sqrt{2^{3n/2}Ft^p}+\sqrt{71\times F^{1/3}R}\right)\left\{\left(E^{1/3}\times F^{5/3}\right)+\left(29\ell^{7/3}s^p\right)+2\sqrt{F^{5/3}E^{1/3}}\sqrt{29\ell^{7/3}s^p}\right\}$$

$$\times\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{Fx^3}+\sqrt{71\times7^{1/3}F^{1/3}r^p}\right)\left\{\left(E^{5/3}y^3\right)+\left(R\ell^{5/3}s^p\right)+2\sqrt{RE^{5/3}\ell^{5/3}y^3s^p}\right\}$$

on multiplying by

$$\left\{\left(7\times2^n\right)\sqrt{2^{n/2}7^{5/3}r^pt^p}\sqrt{2^{3n/2}Ft^p}\left(29\ell^{7/3}s^p\right)\left(F^{1/3}\sqrt{y}\right)\sqrt{71\times7^{1/3}F^{1/3}r^p}\left(R\ell^{5/3}s^p\right)\right\}$$

we get the rational term given by

$$\left\{\left(2^{2n}\times7^2\times29FR\ell^4r^ps^{2p}t^p\right)\sqrt{71y}\right\}$$

VI term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ac^2f\}$

$$=\left(7\times2^n\right)\sqrt{7^{5/3}E^{1/3}r^p}\left(\sqrt{2^{3n/2}Ft^p}+\sqrt{71\times F^{1/3}R}\right)\left(\sqrt{E^{1/3}F^{5/3}}+\sqrt{29\ell^{7/3}s^p}\right)\left(F^{1/3}\sqrt{y}\right)$$

$$\times\left(\sqrt{Fx^3}+\sqrt{71\times7^{1/3}F^{1/3}r^p}\right)\left(\sqrt{z^3\ell^{7/3}s^p}-\sqrt{2^{n/2}F^{5/3}t^p}\right)\left\{\left(E^{5/3}y^3\right)+\left(R\ell^{5/3}s^p\right)+2\sqrt{E^{5/3}y^3}\sqrt{R\ell^{5/3}s^p}\right\}$$

On multiplying by

$$\left\{\left(7\times2^n\right)\sqrt{7^{5/3}E^{1/3}r^p}\sqrt{2^{3n/2}Ft^p}\sqrt{E^{1/3}F^{5/3}}\left(F^{1/3}\sqrt{y}\right)\sqrt{71\times7^{1/3}F^{1/3}r^p}\left(-\sqrt{2^{n/2}F^{5/3}t^p}\right)\left(E^{5/3}y^3\right)\right\}$$

we get

$$\left\{-\left(2^{2n}\times7^2\times E^2F^2F^{2/3}y^3r^pt^p\right)\sqrt{71y}\right\}$$

which is irrational, since F is odd prime.

VII term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a(cd)\}$

$$=\left(-7\times 2^{n}\right)\sqrt{E^{5/3}2^{n/2}t^{p}}\left(\sqrt{2^{3n/2}Ft^{p}}+\sqrt{71\times F^{1/3}R}\right)\left\{\left(E^{1/3}F^{5/3}\right)+\left(29\ell^{7/3}s^{p}\right)+\left(2\sqrt{E^{1/3}F^{5/3}}\sqrt{29\ell^{7/3}s^{p}}\right)\right\}$$
$$\times\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{Fx^{3}}+\sqrt{71\times 7^{1/3}F^{1/3}r^{p}}\right)\left(\sqrt{E^{5/3}y^{3}}+\sqrt{R\ell^{5/3}s^{p}}\right)\left(\sqrt{7^{5/3}y^{3}r^{p}}-\sqrt{3823\ell^{5/3}s^{p}}\right)$$

On multiplying by

$$\left\{\left(-7\times 2^{n}\right)\sqrt{E^{5/3}2^{n/2}t^{p}}\sqrt{2^{3n/2}Ft^{p}}\left(29\ell^{7/3}s^{p}\right)\left(F^{1/3}\sqrt{y}\right)\sqrt{71\times 7^{1/3}F^{1/3}r^{p}}\sqrt{E^{5/3}y^{3}}\sqrt{7^{5/3}y^{3}r^{p}}\right\}$$

we get

$$\left\{-\left(2^{2n}\times 7^{2}\times 29y^{3}FE^{5/3}\ell^{7/3}r^{p}s^{p}t^{p}\right)\sqrt{71y}\right\}$$

which will be irrational, since E is coprime to ℓ, where $\ell = 5$ or 11 or 181, in the equation $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$.

VIII term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a(cd)f\}$

$$=\left(-7E\times 2^{n}\right)\left(\sqrt{2^{3n/2}Ft^{p}}+\sqrt{71\times F^{1/3}R}\right)\left(\sqrt{E^{1/3}F^{5/3}}+\sqrt{29\ell^{7/3}s^{p}}\right)\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{Fx^{3}}+\sqrt{71\times 7^{1/3}F^{1/3}r^{p}}\right)$$
$$\times\left(\sqrt{E^{5/3}y^{3}}+\sqrt{R\ell^{5/3}s^{p}}\right)\left(\sqrt{7^{5/3}y^{3}r^{p}}-\sqrt{3823\ell^{5/3}s^{p}}\right)\left(\sqrt{z^{3}\ell^{7/3}s^{p}}-\sqrt{2^{n/2}F^{5/3}t^{p}}\right)$$

On multiplying by

$$\left\{\left(-7E\times 2^{n}\right)\sqrt{71\times F^{1/3}R}\sqrt{29\ell^{7/3}s^{p}}\left(F^{1/3}\sqrt{y}\right)\sqrt{Fx^{3}}\sqrt{R\ell^{5/3}s^{p}}\left(-\sqrt{3823\ell^{5/3}s^{p}}\right)\sqrt{z^{3}\ell^{7/3}s^{p}}\right\}$$

we get

$$\left\{\left(2^{n}\times 7FER\ell^{4}s^{2p}\right)\sqrt{29x^{3}}\sqrt{71y}\sqrt{3823z^{3}}\right\}$$

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ce^2\}$

$$=\left(7\ell^{2}\right)\sqrt{x^{3}\ell^{7/3}s^{p}}\left\{\left(Ft^{p}\sqrt{2^{3n}}\right)+\left(71F^{1/3}R\right)+2F^{2/3}\sqrt{2^{3n/2}\times 71Rt^{p}}\right\}\left(\sqrt{3823E^{5/3}}+\sqrt{7^{5/3}Rr^{p}}\right)$$
$$\times\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{E^{5/3}y^{3}}+\sqrt{R\ell^{5/3}s^{p}}\right)\left\{\left(E^{1/3}z^{3}\right)+\left(29t^{p}\sqrt{2^{n}}\right)+2\sqrt{E^{1/3}z^{3}}\sqrt{29\times 2^{n/2}t^{p}}\right\}$$

On multiplying by

$$\left\{\left(7\ell^{2}\right)\sqrt{x^{3}\ell^{7/3}s^{p}}\left(2F^{2/3}\sqrt{2^{3n/2}\times 71Rt^{p}}\right)\sqrt{3823E^{5/3}}\left(F^{1/3}\sqrt{y}\right)\sqrt{R\ell^{5/3}s^{p}}\left(2\sqrt{E^{1/3}z^{3}}\sqrt{29\times 2^{n/2}t^{p}}\right)\right\}$$

we get

$$\left\{\left(2^{(n+2)}\times 7\ell^{4}FERs^{p}t^{p}\right)\sqrt{29x^{3}}\sqrt{71y}\sqrt{3823z^{3}}\right\}$$

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{c(ef)\}$

$$=\left(-7\ell^{2}\right)\sqrt{E^{1/3}x^{3}}\left\{\left(Ft^{p}\sqrt{2^{3n}}\right)+\left(71F^{1/3}R\right)+2F^{2/3}\sqrt{2^{3n/2}\times 71Rt^{p}}\right\}\left(\sqrt{3823E^{5/3}}+\sqrt{7^{5/3}Rr^{p}}\right)$$
$$\times\left(F^{1/3}\sqrt{y}\right)\left(\sqrt{E^{5/3}y^{3}}+\sqrt{R\ell^{5/3}s^{p}}\right)\left(\sqrt{E^{1/3}z^{3}}+\sqrt{29\times 2^{n/2}t^{p}}\right)\left(\sqrt{z^{3}\ell^{7/3}s^{p}}-\sqrt{2^{n/2}F^{5/3}t^{p}}\right)$$

On multiplying by

$$\left\{\left(-7\ell^{2}\right)\sqrt{E^{1/3}x^{3}}\left(2F^{2/3}\sqrt{2^{3n/2}\times 71Rt^{p}}\right)\sqrt{3823E^{5/3}}\left(F^{1/3}\sqrt{y}\right)\sqrt{R\ell^{5/3}s^{p}}\sqrt{29\times 2^{n/2}t^{p}}\sqrt{z^{3}\ell^{7/3}s^{p}}\right\}$$

we get

$$\left\{-\left(2^{(n+1)}\times 7\ell^{4}FERs^{p}t^{p}\right)\sqrt{29x^{3}}\sqrt{71y}\sqrt{3823z^{3}}\right\}$$

which is rational.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bce^2\}$

$$=\left(-7\ell^{2}\right)\sqrt{71\times F^{1/3}\ell^{7/3}s^{p}}\left(\sqrt{2^{3n/2}Ft^{p}}+\sqrt{71F^{1/3}R}\right)\left(\sqrt{3823E^{5/3}}+\sqrt{7^{5/3}Rr^{p}}\right)\left(F^{1/3}\sqrt{y}\right)$$
$$\times\left(\sqrt{Rx^{3}}-\sqrt{2^{3n/2}7^{1/3}r^{p}t^{p}}\right)\left(\sqrt{E^{5/3}y^{3}}+\sqrt{R\ell^{5/3}s^{p}}\right)\left\{\left(E^{1/3}z^{3}\right)+\left(29t^{p}\sqrt{2^{n}}\right)+2\sqrt{E^{1/3}z^{3}}\sqrt{29\times 2^{n/2}t^{p}}\right\}$$

Rational part in this term

$$=\left\{\left(-7\ell^{2}\right)\sqrt{71\times F^{1/3}\ell^{7/3}s^{p}}\sqrt{2^{3n/2}Ft^{p}}\sqrt{3823E^{5/3}}\left(F^{1/3}\sqrt{y}\right)\sqrt{Rx^{3}}\sqrt{R\ell^{5/3}s^{p}}\left(2\sqrt{E^{1/3}z^{3}}\sqrt{29\times 2^{n/2}t^{p}}\right)\right\}$$
$$=\left\{-\left(2^{(n+1)}\times 7\ell^{4}FERs^{p}t^{p}\right)\sqrt{29x^{3}}\sqrt{71y}\sqrt{3823z^{3}}\right\}$$

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bc(ef)\}$

$$=\left(7\ell^{2}\right)\sqrt{71\times F^{1/3}E^{1/3}}\left(\sqrt{2^{3n/2}Ft^{p}}+\sqrt{71F^{1/3}R}\right)\left(\sqrt{3823E^{5/3}}+\sqrt{7^{5/3}Rr^{p}}\right)\left(F^{1/3}\sqrt{y}\right)$$

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$$\times\left(\sqrt{Rx^3}-\sqrt{2^{3n/2}7^{1/3}r^pt^p}\right)\left(\sqrt{E^{5/3}y^3}+\sqrt{R\ell^{5/3}s^p}\right)\left(\sqrt{E^{1/3}z^3}+\sqrt{29\times2^{n/2}t^p}\right)\left(\sqrt{z^3\ell^{7/3}s^p}-\sqrt{2^{n/2}F^{5/3}t^p}\right)$$

On multiplying by

$$\left\{\left(7\ell^2\right)\sqrt{71\times F^{1/3}E^{1/3}}\sqrt{2^{3n/2}Ft^p}\sqrt{3823E^{5/3}}\left(F^{1/3}\sqrt{y}\right)\sqrt{Rx^3}\sqrt{R\ell^{5/3}s^p}\sqrt{29\times2^{n/2}t^p}\sqrt{z^3\ell^{7/3}s^p}\right\}$$

we get

$$\left\{\left(2^n\times7\ell^4FERs^pt^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}$$

Sum of all rational part in LHS of equation (8)

$$=\left\{\left(2^n\times7FER\ell^4r^ps^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}\ \text{(combining I \& IV terms)}$$

$$+\left\{\left(2^{2n}\times7^2\times29FR\ell^4r^ps^{2p}t^p\right)\sqrt{71y}\right\}\ \text{(vide V term)}$$

$$+\left\{\left(2^n\times7FER\ell^4s^{2p}\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\right\}\ \text{(vide VIII term)}$$

$$=\left(2^n\times7FER\ell^4s^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\left(r^p+s^p\right)$$

$$+\left(2^{2n}\times7^2\times29FR\ell^4r^ps^{2p}t^p\right)\sqrt{71y}$$

$$=\left(2^n\times7FER\ell^4s^pt^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}$$

$$+\left(2^{2n}\times7^2\times29FR\ell^4r^ps^{2p}t^p\right)\sqrt{71y}\ \left(\because\ r^p+s^p=t^p\right)$$

Sum of all rational part in RHS of equation (8)

$$=\left(2^n\times7FER\ell^4s^pt^p\right)\sqrt{29x^3}\sqrt{71y}\sqrt{3823z^3}\ \text{(combining I to IV terms)}$$

Equating the rational terms on both sides of equation (8), we get

$$\left(2^{2n}\times7^2\times29FR\ell^4r^ps^{2p}t^p\right)\sqrt{71y}=0$$

Dividing both sides by

$$\left\{\left(2^{2n}\times7^2\times29FR\ell^4\right)\sqrt{71y}\right\}$$

we get

$$\left(r^ps^{2p}t^p\right)=0$$

That is, either $r = 0$; or $s = 0$; or $t = 0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$, and proves that only a trivial solution exists in the equation.

III. CONCLUSION

In this proof equation (8) has been obtained from the two transformation equations for $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$. by using the equivalent values of r^p, s^p & t^p, which are substituted in the equation $r^p + s^p = t^p$. Hence the result $rst = 0$, that we get from equation (8) proves that there is no non-zero integer solutions exists in the equation $r^p + s^p = t^p$.

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