A Comprehensible Proof for Fermat's Last Theorem

P. N. Seetharaman

***Abstract:** Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation $A^n + B^n = C^n$ where n is any integer > 2. Taking the proofs of Fermat for the index n = 4, and Euler for n = 3, it is sufficient to prove the theorem for n = p, any prime > 3. We hypothesize that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$ and establish contradiction. Just for supporting the proof in the above equation, we have another equation $x^3 + y^3 = z^3$ Without loss of generality, we assert that both x and y as non-zero integers; z^3 a non-zero integer; z and z^2 irrational. We create transformed equations to the above two equations through parameters, into which we have incorporated the Ramanujan - Nagell equation. Solving the transformed equations we prove the theorem.*

***Keywords:** Transformed Fermat's Equations through Parameters. 2010 Mathematics Subject Classification 2010: 11A–XX.*

I. INTRODUCTION

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of a book that the equation $A^n + B^n = C^n$ has no solution in integers *A*, *B* and *C*, if n is any integer >2. Fermat stated in the margin of the book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index n=4, using infinite descent method [1].

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases [2]. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians [3]. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory [4].

Here we are trying an elementary proof.

II. ASSUMPTIONS

1) We initially hypothesize that all r, s and t are non-zero integers satisfying the equation

$$r^p + s^p = t^p$$

where *p* is any prime > 3, with gcd(*r*, *s*, *t*) = 1 and establish a contradiction in this proof. Both *s* and *t* cannot simultaneously be squares, that is $\sqrt{st}$ will be irrational.

2) Just for supporting the proof in the above equation, we have taken another equation.

$$x^3 + y^3 = z^3; \qquad \gcd(x, y, z^3) = 1$$

Without loss of generality, we can have both *x* and *y* as non-zero integers, z^3 a non-zero integer; both *z* and z^2 irrational. Since we prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of *r*, *s* and *t* we have the choice in having *x*=2×13; *y*=23; $z^3= 23^3+26^3=7^2\times 607$ or *x* = 11,; *y* =53; $z^3 = 11^3 + 53^3 = 8^2 \times 2347$ and so on such that their odd prime factors in *x*, *y* and z^3 could be chosen as coprimes to each of *r*, *s* and *t*.

In the transformation equation the pattern and structures are to be maintained whatever the odd prime factors of *x*, *y* and z^3 are used inside the square roots.

3) We have used the Ramanujan-Nagell equation solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$ in $2^n = 7 + \ell^2$, where *n* is odd and $\ell > 1$.

4) Let *R* be any odd prime, coprime to each of *x*, *y*, z^3, *r*, *s*, *t*, 7 and ℓ. $F = \ell$ and $E = (xyrst)^{3/11}$

Proof. By random trials, we have created the following equations,

$$\left(\frac{a\sqrt{\ell^{7/3}} + b\sqrt{F^{1/3}}}{\sqrt{2^{3n/2}}}\right)^2 + \left(\frac{c\sqrt{7^{5/3}} + d\sqrt{E^{1/3}}}{\sqrt{\ell^{5/3}}}\right)^2 = \left(\frac{e\sqrt{r} + f\sqrt{F^{5/3}}}{\sqrt{7^{1/3}}}\right)^2$$

and

$$\left(a\sqrt{t} - b\sqrt{53}\right)^2 + \left(\frac{c\sqrt{2^{n/2}} - d\sqrt{R}}{\sqrt{2347}}\right)^2 = \left(e\sqrt{11s} - f\sqrt{E^{5/3}}\right)^2 \quad \dots \ (1)$$

to be the transformation equations to $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$, through the parameters called *a*, *b*, *c*, *d*, *e* and *f*. Here *x*= 11, *y* = 53, $z^3 = 11^3 + 53^3 = 8^2 \times 2347$. We have incorporated the Ramanujan-Nagell equation $2^n = 7 + \ell^2$, limited to *n* as an odd integer and $\ell > 1$. *F*, *E* and *R* be distinct odd primes.

From equation (1), we get

$$a\sqrt{\ell^{7/3}} + b\sqrt{F^{1/3}} = \sqrt{2^{3n/2}x^3} \quad \dots \ (2)$$

$$a\sqrt{t} - b\sqrt{53} = \sqrt{r^p} \quad \dots \ (3)$$

$$c\sqrt{7^{5/3}} + d\sqrt{E^{1/3}} = \sqrt{y^3\ell^{5/3}} \quad \dots \ (4)$$

$$c\sqrt{2^{n/2}} - d\sqrt{R} = \sqrt{2347s^p} \quad \dots \ (5)$$

$$e\sqrt{r} + f\sqrt{F^{5/3}} = \sqrt{7^{1/3}z^3} \quad \dots \ (6)$$

and

$$e\sqrt{11s} - f\sqrt{E^{5/3}} = \sqrt{t^p} \quad \dots \ (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

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*Correspondence Author(s)
P. N Seetharaman*, (Retired Executive Engineer, Energy Conservation Cell), Tamil Nadu State Electricity Board, Tamil Nadu, India. Email ID: palamadaiseetharaman@gmail.com, ORCID ID: 0000-0002-4615-1280

$$a=\left(\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right)\Big/\left(\sqrt{F^{1/3}t}+\sqrt{53\times \ell^{7/3}}\right)$$
$$b=\left(\sqrt{2^{3n/2}x^3t}-\sqrt{\ell^{7/3}r^p}\right)\Big/\left(\sqrt{F^{1/3}t}+\sqrt{53\times \ell^{7/3}}\right)$$
$$c=\left(\sqrt{Ry^3\ell^{5/3}}+\sqrt{2347E^{1/3}s^p}\right)\Big/\left(\sqrt{E^{1/3}\times 2^{n/2}}+\sqrt{R\times 7^{5/3}}\right)$$
$$d=\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times s^p7^{5/3}}\right)\Big/\left(\sqrt{E^{1/3}\times 2^{n/2}}+\sqrt{R\times 7^{5/3}}\right)\qquad e=\left(\sqrt{7^{1/3}E^{5/3}z^3}+\sqrt{F^{5/3}t^p}\right)\Big/\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)$$
and $$f=\left(\sqrt{11\times 7^{1/3}z^3s}-\sqrt{rt^p}\right)\Big/\left(\sqrt{E^{5/3}r}+\sqrt{11\times F^{5/3}s}\right)$$

From (2) & (5), we have

$$\sqrt{2^{n/2}}\times\sqrt{2^{3n/2}}=\left(a\sqrt{\ell^{7/3}}+b\sqrt{F^{1/3}}\right)\left(\sqrt{2347s^p}+d\sqrt{R}\right)\Big/\left(c\sqrt{x^3}\right)$$

i.e., $$2^n=\left\{(a)\sqrt{2347s^p\ell^{7/3}}+(ad)\sqrt{R\times\ell^{7/3}}+(b)\sqrt{2347F^{1/3}s^p}+(bd)\sqrt{F^{1/3}R}\right\}\Big/\left(c\sqrt{x^3}\right)$$

From (4) & (6), we get

$$\sqrt{7^{1/3}}\times\sqrt{7^{5/3}}=\left(\sqrt{y^3\ell^{5/3}}-d\sqrt{E^{1/3}}\right)\left(e\sqrt{r}+f\sqrt{F^{5/3}}\right)\Big/\left(c\sqrt{z^3}\right)$$

i.e., $$7=\left\{(e)\sqrt{y^3r\ell^{5/3}}+(f)\sqrt{F^{5/3}y^3\ell^{5/3}}-(de)\sqrt{E^{1/3}r}-(df)\sqrt{F^{5/3}E^{1/3}}\right\}\Big/\left(c\sqrt{z^3}\right)$$

From (2) & (4), we get

$$\sqrt{\ell^{7/3}}\times\sqrt{\ell^{5/3}}=\left(\sqrt{2^{3n/2}x^3}-b\sqrt{F^{1/3}}\right)\left(c\sqrt{7^{5/3}}+d\sqrt{E^{1/3}}\right)\Big/\left(a\sqrt{y^3}\right)$$

i.e., $$\ell^2=\left\{(c)\sqrt{2^{3n/2}\times 7^{5/3}x^3}+(d)\sqrt{E^{1/3}\times 2^{3n/2}x^3}-(bc)\sqrt{F^{1/3}\times 7^{5/3}}-(bd)\sqrt{F^{1/3}E^{1/3}}\right\}\Big/\left(a\sqrt{y^3}\right)$$

Substituting the above equivalent values of 2^n, 7 and ℓ^2 in Ramanujan-Nagell equation $2^n = 7 + \ell^2$ after multiplying both sides by $\left\{(ac)\sqrt{x^3y^3z^3}\right\}$, we get

$$\left[(a)\sqrt{y^3z^3}\right]\left\{(a)\sqrt{2347\times s^p\ell^{7/3}}+(ad)\sqrt{R\ell^{7/3}}+(b)\sqrt{2347F^{1/3}s^p}+(bd)\sqrt{F^{1/3}R}\right\}$$
$$=\left\{(a)\sqrt{x^3y^3}\right\}\left\{(e)\sqrt{y^3r\ell^{5/3}}+(f)\sqrt{y^3F^{5/3}\ell^{5/3}}-(de)\sqrt{E^{1/3}r}-(df)\sqrt{F^{5/3}E^{1/3}}\right\}$$
$$+\left\{(c)\sqrt{x^3z^3}\right\}\left\{(c)\sqrt{2^{3n/2}\times 7^{5/3}x^3}+(d)\sqrt{2^{3n/2}E^{1/3}x^3}-(bc)\sqrt{F^{1/3}\times 7^{5/3}}-(bd)\sqrt{F^{1/3}E^{1/3}}\right\}\quad (8)$$

Our aim is to workout all rational terms in equation (8) after multiplying both sides by

$$\left\{\left(\sqrt{53\times\ell^{7/3}}+\sqrt{F^{1/3}t}\right)^2\left(\sqrt{7^{5/3}R}+\sqrt{E^{1/3}\times 2^{n/2}}\right)^2\left(\sqrt{E^{5/3}r}+\sqrt{11\times F^{5/3}s}\right)\right\}$$

for freeing from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* and again multiplying both sides by

$$\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$

for getting some rational terms as worked out below, term by term.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2\}$

$$=\sqrt{y^3z^3}\sqrt{2347\times s^p\ell^{7/3}}\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\left\{\left(7^{5/3}R\right)+\left(E^{1/3}\sqrt{2^n}\right)+2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right\}$$
$$\times\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left\{\left(53x^3\sqrt{2^{3n}}\right)+\left(F^{1/3}r^p\right)+\left(2\sqrt{53\times 2^{3n/2}x^3}\sqrt{F^{1/3}r^p}\right)\right\}$$

On multiplying by

$$\left\{\sqrt{y^3z^3}\sqrt{2347\times s^p\ell^{7/3}}\sqrt{E^{5/3}r}\left(2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(53x^3\sqrt{2^{3n}}\right)\right\}$$

We get

$$\left\{2^{2n+1}\times 7\times 53Rx^3\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational since $F=\ell$ and $E^{11/3}(xyrst)$.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2d\}$ is

$$=\sqrt{y^3z^3}\sqrt{R\ell^{7/3}}\left(\sqrt{E^{1/3}\times 2^{n/2}}+\sqrt{7^{5/3}R}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left\{\left(53x^3\sqrt{2^{3n}}\right)+\left(F^{1/3}r^p\right)+2\sqrt{53\times 2^{3n/2}x^3}\sqrt{F^{1/3}r^p}\right\}\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times s^p7^{5/3}}\right)$$

On multiplying by

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$$\left\{\sqrt{y^3z^3}\sqrt{R\ell^{7/3}}\sqrt{E^{1/3}\times 2^{n/2}}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(53x^3\sqrt{2^{3n}}\right)\left(-\sqrt{2347\times s^p 7^{5/3}}\right)\right\}$$

We get

$$\left\{-\left(2^{2n}\times 7\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ab\}$

$$=\sqrt{y^3z^3}\sqrt{2347\times F^{1/3}s^p}\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\left\{\left(7^{5/3}R\right)+\left(E^{1/3}\sqrt{2^n}\right)+2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right\}$$
$$\times\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right)\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)$$

On multiplying by

$$\left\{\sqrt{y^3z^3}\sqrt{2347\times F^{1/3}s^p}\sqrt{E^{5/3}r}\left(2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times 2^{3n/2}x^3}\sqrt{2^{3n/2}x^3t}\right\}$$

We get

$$\left\{2^{2n+1}\times 7\times FRx^3t\sqrt{53y^3}\sqrt{2347z^3}\sqrt{E^{11/3}xrs^p}\right\}$$

Which is irrational, since $E^{11/3}=xyrst$, and t is coprime to $y = 53$. Otherwise, we have the choice of assigning alternative value for y. (see assumptions)

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)d\}$

$$=\sqrt{F^{1/3}Ry^3z^3}\left(\sqrt{2^{n/2}\times E^{1/3}}+\sqrt{R\times 7^{5/3}}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left(\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right)\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times 7^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\sqrt{F^{1/3}Ry^3z^3}\sqrt{2^{n/2}\times E^{1/3}}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times 2^{3n/2}x^3}\sqrt{2^{3n/2}x^3t}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\right\}$$

We get

$$\left\{-\left(2^{2n}\times 7FRx^3t\right)\sqrt{53y^3}\sqrt{2347z^3}\sqrt{E^{11/3}xrs^p}\right\}$$

Which is irrational.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ae\}$

$$=\left(y^3\sqrt{rx^3\ell^{5/3}}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left\{\left(E^{1/3}\sqrt{2^n}\right)+\left(7^{5/3}R\right)+2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right\}\left(\sqrt{7^{1/3}E^{5/3}z^3}+\sqrt{F^{5/3}t^p}\right)$$

On multiplying by

$$\left\{\left(y^3\sqrt{rx^3\ell^{5/3}}\right)\sqrt{53\times\ell^{7/3}}\sqrt{F^{1/3}r^p}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right)\sqrt{F^{5/3}t^p}\right\}$$

We get

$$\left\{\left(2\times 7ER\ell^2x^2y^3\right)\sqrt{(rt)^{p+1}}\sqrt{53\times F^{11/3}2^n}\right\}$$

Which is irrational.

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{af\}$

$$=\left(y^3\sqrt{F^{5/3}x^3\ell^{5/3}}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\left\{\left(7^{5/3}R\right)+\left(E^{1/3}\sqrt{2^n}\right)+2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right\}\left(\sqrt{11\times 7^{1/3}z^3s}-\sqrt{rt^p}\right)$$

On multiplying by

$$\left\{\left(y^3\sqrt{F^{5/3}x^3\ell^{5/3}}\right)\sqrt{53\times\ell^{7/3}}\sqrt{F^{1/3}r^p}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(2\sqrt{7^{5/3}R}\sqrt{2^{n/2}E^{1/3}}\right)\left(-\sqrt{rt^p}\right)\right\}$$

We get

$$\left\{-\left(2\times 7ER\ell^2x^3y^3\right)\sqrt{(rt)^{p+1}}\sqrt{53\times F^{11/3}2^n}\right\}$$

Which is irrational.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ade\}$

$$=\left(-\sqrt{E^{1/3}x^3y^3r}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{1/3}\times 2^{n/2}}+\sqrt{R\times 7^{5/3}}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left\{\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right\}\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times 7^{5/3}s^p}\right)\left(\sqrt{7^{1/3}E^{5/3}z^3}+\sqrt{F^{5/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{E^{1/3}x^3y^3r}\right)\sqrt{53\times\ell^{7/3}}\sqrt{R\times 7^{5/3}}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times 2^{3n/2}x^3}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\sqrt{7^{1/3}E^{5/3}z^3}\right\}$$ We get

$$\left\{\left(2^n\times 7^2\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational, since $z^3 = 8^2\times 2347$; $F=\ell$ and $E^{11/3} = (xyrst)$.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(adf\}$

$$=\left(-\sqrt{x^3y^3}\right)\sqrt{F^{5/3}E^{1/3}}\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{1/3}\times 2^{n/2}}+\sqrt{R\times 7^{5/3}}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left\{\sqrt{53\times 2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right\}\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times 7^{5/3}s^p}\right)\left(\sqrt{11\times 7^{1/3}z^3s}-\sqrt{rt^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{x^3y^3}\right)\sqrt{F^{5/3}E^{1/3}}\sqrt{53\times\ell^{7/3}}\sqrt{R\times 7^{5/3}}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times 2^{3n/2}x^3}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\sqrt{11\times 7^{1/3}z^3s}\right\}$$

We get

$$\left\{\left(2^n\times 7^2\times 53ERx^3\right)\sqrt{11x}\sqrt{s^{p+1}}\sqrt{F^{10/3}\ell^{7/3}ty^3}\sqrt{2347z^3}\right\}$$

Which will be irrational since $F=\ell$.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{c^2\}$

$$=\left(x^3\right)\sqrt{2^{3n/2}\times 7^{5/3}z^3}\left\{\left(F^{1/3}t\right)+\left(53\ell^{7/3}\right)+2\sqrt{53\ell^{7/3}}\sqrt{F^{1/3}t}\right\}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\left\{\left(Ry^3\ell^{5/3}\right)+\left(2347E^{1/3}s^p\right)+2\sqrt{2347E^{1/3}s^p}\sqrt{Ry^3\ell^{5/3}}\right\}$$

On multiplying by

$$\left(x^3\right)\sqrt{2^{3n/2}\times 7^{5/3}z^3}\left(53\ell^2\sqrt{\ell^{2/3}}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{E^{5/3}r}\left(2\sqrt{2347E^{1/3}s^p}\sqrt{Ry^3\ell^{5/3}}\right)$$

We get

$$\left\{\left(2^{n+1}\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{cd\}$ is

$$=\left(x^3\right)\sqrt{2^{3n/2}E^{1/3}z^3}\left\{\left(F^{1/3}t\right)+\left(53\times\ell^{7/3}\right)+\left(2\sqrt{53F^{1/3}\ell^{7/3}t}\right)\right\}\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)$$
$$\times\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\left(\sqrt{R\ell^{5/3}y^3}+\sqrt{2347E^{1/3}s^p}\right)\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times 7^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(x^3\right)\sqrt{2^{3n/2}E^{1/3}z^3}\left(53\ell^2\sqrt{\ell^{2/3}}\right)\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{R\ell^{5/3}y^3}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\right\}$$

We get

$$\left\{-\left(2^n\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bc^2\}$ is

$$=\left(-\sqrt{7^{5/3}F^{1/3}x^3z^3}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$
$$\times\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)\left\{\left(Ry^3\ell^{5/3}\right)+\left(2347E^{1/3}s^p\right)+\left(2\sqrt{2347E^{1/3}s^p}\right)\sqrt{R\ell^{5/3}y^3}\right\}$$

On multiplying by

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$$\left\{\left(-\sqrt{7^{5/3}F^{1/3}x^3z^3}\right)\sqrt{F^{1/3}t}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{2^{3n/2}x^3t}\left(\left(2\sqrt{2347E^{1/3}s^p}\right)\sqrt{R\ell^{5/3}y^3}\right)\right\}$$

We get

$$\left\{-\left(2^{n+1}\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)\}$ is

$$=\left(-\sqrt{F^{1/3}E^{1/3}x^3z^3}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}$$

$$\times\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)\left(\sqrt{R\ell^{5/3}y^3}+\sqrt{2347E^{1/3}s^p}\right)\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times 7^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{F^{1/3}E^{1/3}x^3z^3}\right)\sqrt{F^{1/3}t}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{2^{3n/2}x^3t}\sqrt{R\ell^{5/3}y^3}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\right\}$$

We get

$$\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is ration al

Sum of all rational terms in LHS of equation (8)

$$=\left\{\left(2^{2n}\times 7\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining I \& II terms)}$$

Sum of all rational terms in RHS of equation (8) is

$$=\left\{\left(2^n\times 7^2\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(vide III term)}$$

$$+\left\{\left(2^n\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining V \& VI terms)}$$

$$-\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining VII \& VIII terms)}$$

$$=\left\{\left(2^{2n}\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\left(\because\ 7+\ell^2=2^n\right)$$

$$-\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Equating the rational terms on both sides of equation (8),
We get

$$-\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}=0$$

Dividing both sides by

$$\left\{-\left(2^n\times 7\times FRx^3\right)\sqrt{2347z^3}\right\}$$

We get

$$\left(t\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right)=0$$

i.e., either $r=0$ or $s=0$ or $t=0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p+s^p=t^p$, with p any prime > 3, thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

Since equation (8) in this proof has been derived directly from the transformation equations the result that we have obtained on equating the rational terms on both sides of equation (8) should reflect on the Fermat's Equation $r^p+s^p=t^p$, thus proving that only a trivial solution exists in the equation $r^p+s^p=t^p$.

The only main hypothesis that we made in this proof, namely, r, s and t are non-zero integers has been shattered by the result $rst=0$, thus proving the theorem.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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AUTHOR'S PROFILE

P. N. Seetharaman, (Date of Birth: 7th December 1945) had studied B.Sc (Mathematics (1963-66) at St.Joseph College in Tiruchirappalli in Tamilnadu, and he did his B.E (Electrical Engineering) in College of Engineering. Guindy,at Chennai (1966-69), and joined as an Engineer in Tamilnadu State Electricity Board. He had served as an operator for running the Mettur Tunnel Hydro Power Station (4×50 MW) for ten years from 1986 1996., which was feeding Power through 230KV interstate grid. He had also worked as an engineer, in the distribution of power. Finally he worked as Executive Engineer in Tamilnadu Electricity Board Headquarters office in Chennai in Research and Development Wing, in Energy Conservation Cell, for doing Energy Conservation by High Tenstion factories , drawing Power from Tamilnadu. After his retirement in 2002, he learned number theory as his special interest in mathematics from Anna Centenary Library at Chennai for two decades by self-study, and learned elementary number theory and Fermat's Last Theorem.