# Some Algorithms of Graph Theory in Cryptology 


#### Abstract

The inventive use of concepts from Graph Theory plays a significant role in hiding the original Plain-text for resulting in a significantly safe data transfer. In this work, the tree traversal algorithms like Inorder, Preorder, Postorder, Kruskal's algorithm for making minimal spanning tree and the modified graph labelling scheme of graceful labelling allowing repetition of exactly one vertex label for certain graphs, have been employed to create highly hidden Cipher-texts. Encryption and decryption algorithms for all these methods are being presented in this work.


Keywords: Modified Graceful Labelling, Tree Traversal Algorithms, Expression Tree, Modified Ordered Rooted Tree, Basic-Cipher-text, Cipher-Matrix (of different kinds), Encryption-Array.

## I. INTRODUCTION

Thhe tree traversal algorithms: Inorder, Preorder and Postorder and their respective notations along with the trees can be found in Kolman, Busby and Ross[1]. Katz [2] is a survey which gives the basic definitions of binary tree encryption (BTE), some recent applications to forward secure encryption, identity- based and hierarchical identitybased encryption, chosen cipher-text security and adaptively-secure encryption. Krishnaa and Dulawat [3] have given the modified graceful labelling method allowing repetition of exactly one repeated vertex label. Shanmugam [4] presents a technique for security using a combination of Caesar Cipher and graph traversal. Sivakumar, Humshavarthini, Jayasree and Eswaran [5] present encryption and decryption using ASCII values of characters to obtain first level encryption and then the Binary Tree Traversal (BTT)used in the second level of encryption for achieving permutation. Krishnaa [6] has given the application of inner magic and inner antimagic labeling with both encryption and decryption algorithms using the Wheel graph whereas several specific graphs with their encryption and decryption algorithms have been given in Krishnaa [7]. Krishnaa [8] presents the cryptological application employing Lattices using certain concepts of Lattices of Discrete Mathematics. The concept of Basic Cipher Text has been introduced in Krishnaa and Gurjar [9] to further increase the hiding capacity of the final Cipher texts to be developed subsequently, or the option of using the Basic-Cipher-Text as the final Cipher Text itself.

[^0]In the subsection A, the tree traversal algorithms: Inorder, Preorder and Postorder, both with and without expression trees have been used to create the Cipher-texts. In subsection B, the modified graceful labelling allowing repetition of exactly one vertex label for binary tree and the doublestar graphs, and in subsection C the Kruskal's algorithm for making the minimal spanning tree using the concept of modified ordered rooted tree are being presented for creating highly hidden Cipher-texts. For all these concepts, encryption and decryption algorithms have been presented.

## II. MAIN RESULTS

## A. Tree Traversal Algorithms

A Basic-Cipher-text using the tree traversal algorithms has been created to provide added security in hiding the Plain-text. The process of visiting every vertex of a tree in a certain order is called tree traversal. It is of 3 kinds and mainly performed on binary trees. In a binary tree, each vertex has at most two (left and right) offsprings. The 3 tree traversal algorithms are as follows:

## Preorder:

1. Visit the root
2. Search the left subtree if it exists
3. Search the right subtree if it exists.

A Preorder traversal yields a Prefix notation.

## Inorder:

1. Search the left subtree if it exists
2. Visit the root
3. Search the right subtree if it exists

An Inorder traversal yields an Infix notation.

## Postorder:

1. Search the left subtree if it exists
2. Search the right subtree if it exists
3. Visit the root.

A Postorder traversal yields a Postfix notation.
a. Tree Traversal Algorithms Without Expression Trees
An expression tree represents a mathematical expression. In this section a case of making Cipher-texts is demonstrated without using an expression tree.


Figure 1: Tree used without Mathematical Expression for Preorder Traversal

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## Encryption:

1. The Plain-text: GRAPHTHEORY is assigned to the vertices of the graph in Figure 1 in Prefix notation (abc $\mathrm{g} \mathrm{h} \mathrm{i} \mathrm{d} \mathrm{k} \mathrm{e} \mathrm{j} \mathrm{f)} \mathrm{where} \mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3 \ldots$ are the names of the vertices, where by the Preorder Traversal, the Basic-Cipher-text obtained is: G R P E O H A T H R Y and the assignments are as shown below:

$$
\begin{aligned}
& \mathrm{G}=\mathrm{a}=\mathrm{v} 1 \\
& \mathrm{R}=\mathrm{b}=\mathrm{v} 2 \\
& \mathrm{P}=\mathrm{c}=\mathrm{v} 4 \\
& \mathrm{E}=\mathrm{g}=\mathrm{v} 8 \\
& \mathrm{O}=\mathrm{h}=\mathrm{v} 9 \\
& \mathrm{H}=\mathrm{i}=\mathrm{v} 5 \\
& \mathrm{~A}=\mathrm{d}=\mathrm{v} 3 \\
& \mathrm{~T}=\mathrm{k}=\mathrm{v} 6 \\
& \mathrm{H}=\mathrm{e}=\mathrm{v} 7 \\
& \mathrm{R}=\mathrm{j}=\mathrm{v} 10 \\
& \mathrm{Y}=\mathrm{f}=\mathrm{v} 11
\end{aligned}
$$

2. The Cipher-text H T T M X M D Z O B J is calculated (from top to bottom) by adding the number " $i$ " in "vi" to the letters of the Basic-Cipher-text shown below in the Cipher-Matrix (only the Cipher-text needs to be sent to

|  | ¢ | b | c | V | e | f | g | h | 1 | J | k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vertex a | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| b | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| c | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| d | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| e | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| f | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| g | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| h | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| J | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| k | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

4. Send the Adjacency Matrix, Cipher-text H T T M X M D Z O B J, Cipher-Matrix to the receiver. The vertices in the Adjacency Matrix can be labelled as $\mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3, \ldots$ or a,b,c,....

## Decryption:

1. Arrange vi's in the Cipher-Matrix in ascending order.
2. The original Plain-text is calculated by the reverse operation i.e., subtraction as follows:
the receiver; calculations of the Cipher-text is only for demonstration purpose)

## Cipher-Matrix

| $\mathrm{G}+1=\mathrm{H}$ | a | v 1 |
| :--- | :--- | :--- |
| $\mathrm{R}+2=\mathrm{T}$ | b | v 2 |
| $\mathrm{P}+4=\mathrm{T}$ | c | v 4 |
| $\mathrm{E}+8=\mathrm{M}$ | g | v 8 |
| $\mathrm{O}+9=\mathrm{X}$ | h | v 9 |
| $\mathrm{H}+5=\mathrm{M}$ | i | v 5 |
| $\mathrm{~A}+3=\mathrm{D}$ | d | v 3 |
| $\mathrm{~T}+6=\mathrm{Z}$ | k | v 6 |
| $\mathrm{H}+7=\mathrm{O}$ | e | v 7 |
| $\mathrm{R}+10=\mathrm{B}$ | j | v 10 |
| $\mathrm{Y}+11=\mathrm{J}$ | f | v 11 |

3. Adjacency Matrix: It is a matrix which can represent the graph in matrix form such that its matrix entry $A_{a b}=1$ if there is an edge between vertices $a$ and $b$ else $A_{a b}$ $=0$.
The Adjacency Matrix for the graph in Figure 1 is given below:
$\mathrm{H}-1=\mathrm{G}, \mathrm{T}-2=\mathrm{R}, \mathrm{D}-3=\mathrm{A}, \mathrm{T}-4=\mathrm{P}, \mathrm{M}-5=\mathrm{H}, \mathrm{Z}-$ $6=\mathrm{T}, \mathrm{O}-7=\mathrm{H}, \mathrm{M}-8=\mathrm{E}, \mathrm{X}-9=\mathrm{O}$,
$\mathrm{B}-10=\mathrm{R}, \mathrm{J}-11=\mathrm{Y}$, therefore yielding the original Plaintext: GRAPHTHEORY.

b. Using Expression Trees in Inorder and Postorder traversals to make Cipher-texts:

## Encryption:

1. Make the following assignments of the Plain-text: COUNTRIES to the vertices of the expression tree of Figure 2 as follows:

$$
\begin{aligned}
\mathrm{v} 1 & =\mathrm{C}=\mathrm{x} \\
\mathrm{v} 2 & =\mathrm{O}=- \\
\mathrm{v} 3 & =\mathrm{U}=\mathrm{a} \\
\mathrm{v} 4 & =\mathrm{N}=\mathrm{b} \\
\mathrm{v} 5 & =\mathrm{T}=+ \\
\mathrm{v} 6 & =\mathrm{R}=\mathrm{c} \\
\mathrm{v} 7 & =\mathrm{I}=/ \\
\mathrm{v} 8 & =\mathrm{E}=\mathrm{d} \\
\mathrm{v} 9 & =\mathrm{S}=\mathrm{e}
\end{aligned}
$$



Figure 2: Expression Tree for Postorder and Inorder Traversal

## Postfix-Cipher-Matrix

| $\mathrm{U}+3=\mathrm{X} \mathrm{a}$ | v 3 |
| :--- | :--- |
| $\mathrm{~N}+4=\mathrm{R} \mathrm{b}$ | v 4 |
| $\mathrm{O}+2=\mathrm{Q}-$ | v 2 |
| $\mathrm{R}+6=\mathrm{X} \mathrm{c}$ | v 6 |
| $\mathrm{E}+8=\mathrm{M} \mathrm{d}$ | v 8 |
| $\mathrm{~S}+9=\mathrm{B} \mathrm{e}$ | v 9 |
| $\mathrm{I}+7=\mathrm{P} /$ | v 7 |
| $\mathrm{~T}+5=\mathrm{Y}+$ | v 5 |
| $\mathrm{C}+1=\mathrm{D} \mathrm{x}$ | v 1 |

4. Send the Cipher-texts XRQXMBPYD (by Postfix notation) and XQRDXYMPB (by Infix notation), the respective Postfix-Cipher-Matrix and Infix-Cipher-Matrix and the Adjacency Matrix of the tree to the receiver. It is to be noted that the first column in each Cipher-Matrix is shown for calculation purpose, only Cipher-text is to be sent to the receiver.

## Decryption:

1. Perform a Preorder Traversal on the original tree obtained by the Adjacency Matrix yielding the Prefix notation: $x-a b+c / d e$. It is to be noted that the Preorder traversal of the expression i.e., $x-a b+c / d e$ is the ascending order of the vi's in both Postfix-Cipher- Matrix and Infix-Cipher-matrix.
2. Arrange the vi's in ascending order
3. Subtract the number "i" of "vi" from the respective letter of the Cipher-text to get the original Plain-text.
4. Two Cipher-texts are obtained as shown below:
(a) Postorder Traversal yielding Postfix notation: $\mathrm{ab}-\mathrm{c}$ $\mathrm{de} /+\mathrm{x}$ and obtain the Cipher-text by adding the " $i$ " number in vertex "vi" to the letters of the Basic-Cipher-text UNORESITC obtained by the Postorder traversal
(b) Inorder Traversal yielding Infix notation: $\mathrm{a}-\mathrm{b} \mathrm{x} \mathrm{c}+$ d/e and obtain the

Cipher-text by adding the "i" number in vertex "vi" to the letters of the Basic-Cipher-text UONCRTEIS obtained by the Inorder traversal.
3. The two Cipher-texts are shown in the following matrices in their right most columns from top to bottom namely: XRQXMBPYD and XQRDXYMPB (only the Cipher-text needs to be sent to the receiver; calculations of the Cipher-text is only for demonstration purpose)

## Infix-Cipher-Matrix

| $\mathrm{U}+3=\mathrm{X}$ | a | v 3 |
| :--- | :--- | :--- |
| $\mathrm{O}+2=\mathrm{Q}$ | - | v 2 |
| $\mathrm{~N}+4=\mathrm{R}$ | b | v 4 |
| $\mathrm{C}+1=\mathrm{D}$ | x | v 1 |
| $\mathrm{R}+6=\mathrm{X}$ | c | v 6 |
| $\mathrm{~T}+5=\mathrm{Y}$ | + | v 5 |
| $\mathrm{E}+8=\mathrm{M}$ | d | v 8 |
| $\mathrm{I}+7=\mathrm{P} /$ | v 7 |  |
| $\mathrm{~S}+9=\mathrm{B}$ | e | v 9 |

(i)Postfix: $\mathrm{D}-1=\mathrm{C}, \mathrm{Q}-2=\mathrm{O}, \mathrm{X}-3=\mathrm{U}, \mathrm{R}-4=\mathrm{N}, \mathrm{Y}-$ $5=\mathrm{T}, \mathrm{X}-6=\mathrm{R}, \mathrm{P}-7=\mathrm{I}, \mathrm{M}-8=\mathrm{E}$,
B-9 $=$ S. Therefore, the original Plain-text: COUNTRIES is obtained.
(ii)Infix : $\mathrm{D}-1=\mathrm{C}, \mathrm{Q}-2=\mathrm{O}, \mathrm{X}-3=\mathrm{U}, \mathrm{R}-4=\mathrm{N}, \mathrm{Y}-5$ $=\mathrm{T}, \mathrm{X}-6=\mathrm{R}, \mathrm{P}-7=\mathrm{I}, \mathrm{M}-8=\mathrm{E}, \mathrm{B}-9=\mathrm{S}$. Therefore, the original Plain-text: COUNTRIESis obtained.

Another option can be to have the original Plain-text message in Postfix notation and the Cipher-texts in Prefix and Infix notation as the suitability of Prefix and Postfix notations is more for the computer, both being free of using parentheses while the Infix notation is ambiguous without parentheses and more suitable for human use.

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In case of the expression not being so important, the Infix notation can also be used to stand for the original Plain-text message and have the Cipher-texts in Prefix and Postfix notations and proceed similarly.
B. Binary Tree and Doublestar with Modified Graceful Labelling Allowing Repetition of Exactly One Vertex Label:
Graph Labelling: It is an assignment of labels (numbers) to vertices or edges such that the induced edge or vertex labels follow a certain pattern.
Graceful Labelling: For a graph $G(p, q)$ with $p$ number of vertices and q number of edges, a graceful labelling is given by assigning the vertices with $0,1,2, \ldots, q$ such that the induced edge labels are given by $\mathrm{f}^{*}(\mathrm{x}, \mathrm{y})=|\mathrm{f}(\mathrm{x})-\mathrm{f}(\mathrm{y})|$ where $f(x)$ and $f(y)$ are the vertex labels of the vertices $x$ and $y$ and $f *$ is injective.
Modified Graceful Labelling: Given in [3], this allows repetition of exactly one vertex label and the induced edge label of 0 is allowed.
Binary Tree: The Plain-text for creating Cipher-text for this graph is: COUNTRY. The vertex labels are those of the modified graceful labelling allowing repetition of exactly one vertex label.

## Encryption (Binary Tree):

1. The vertices in the Binary Tree of Figure 3 are given the following assignments exhibiting the modified graceful labelling as given in [3] (right-most column gives the vertex labels of the modified graceful labelling):

$$
\begin{aligned}
\mathrm{v} 1 & =\mathrm{C}=0 \\
\mathrm{v} 2 & =\mathrm{O}=5 \\
\mathrm{v} 3 & =\mathrm{U}=0 \\
\mathrm{v} 4 & =\mathrm{N}=1 \\
\mathrm{v} 5 & =\mathrm{T}=4 \\
\mathrm{v} 6 & =\mathrm{R}=3 \\
\mathrm{v} 7 & =\mathrm{Y}=2
\end{aligned}
$$



Figure 3: Binary Tree
2. The Cipher-text CTUPXUA is calculated by adding the respective vertex labels namely $0,5,0,1,4,3,2$ of the vertices v1, v2, v3, ..., v7 to the letters of the Plaintext COUNTRY as follows:
$\mathrm{C}+0=\mathrm{C}, \mathrm{O}+5=\mathrm{T}, \mathrm{U}+0=\mathrm{U}, \mathrm{N}+1=\mathrm{P}, \mathrm{T}+4=\mathrm{X}, \mathrm{R}+$ $3=\mathrm{U}, \mathrm{Y}+2=$ A thus, yielding the Cipher-text CTUPXUA
3. Make the Cipher-Matrix-Binary Tree as follows:

Cipher-Matrix-Binary Tree

| 2 | v7 | A |
| :--- | :--- | :--- |
| 4 | v5 | X |
| 3 | v6 | U |
| 5 | v2 | T |
| 1 | v4 | P |
| 0 | v1 | C |
| 0 | v3 | U |

4. Send the Cipher-text, Cipher-Matrix-Binary-Tree to the receiver.

## Decryption (Binary Tree):

1. In the order of $\mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3, \ldots \mathrm{v} 7$ or the vertex labels of the modified graceful labelling 0501
432 of the Binary Tree, associate letters of the Cipher-text with the vertex labels as follows: $\mathrm{C}=0, \mathrm{~T}=5, \mathrm{U}=0, \mathrm{P}=$ $1, \mathrm{X}=4, \mathrm{U}=3, \mathrm{~A}=2$
2. Arrange the vertex labels 0501432 in the ascending order $\mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3, \ldots \mathrm{v} 7$ (these vertices
and the respective vertex labels are of the modified graceful labelling of the Binary Tree)
3. Subtract the vertex labels from the letters of the Cipher-text to obtain the Plain-text as shown below:
$\mathrm{C}-0=\mathrm{C}, \mathrm{T}-5=\mathrm{O}, \mathrm{U}-0=\mathrm{U}, \mathrm{P}-1=\mathrm{N}, \mathrm{X}-4=\mathrm{T}, \mathrm{U}-3$
$=\mathrm{R}, \mathrm{A}-2=\mathrm{Y}$ therefore yielding the original Plain-text COUNTRY.
Doublestar: this graph has been mentioned and explored in [3] for studying the effect of repeating a vertex label in the various graph labelling schemes. It is a kind of tree. In this case, an additional binary tree has been taken to generate the initial Basic-Cipher-text for an added secrecy in hiding of the original Plain-text message by Inorder traversal. Doublestar also exhibits a modified graceful labelling as per [3].


Figure 4: Doublestar
Plain-text is COUNTRY

## Encryption (Doublestar):



Figure 5: Inorder traversal to get Basic-Cipher-text: $Y$ TRONCU for the Doublestar

1. The vertices in the Doublestar of Figure $\mathbf{4}$ are given the following assignments (right-most column gives the vertex labels of the modified graceful labelling):

$$
\begin{aligned}
\mathrm{v} 1 & =\mathrm{C}=0 \\
\mathrm{v} 2 & =\mathrm{O}=2 \\
\mathrm{v} 3 & =\mathrm{N}=4 \\
\mathrm{v} 4 & =\mathrm{U}=0 \\
\mathrm{v} 5 & =\mathrm{T}=3 \\
\mathrm{v} 6 & =\mathrm{Y}=1 \\
\mathrm{v} 7 & =\mathrm{R}=5
\end{aligned}
$$

2. Use the adjacency matrices of Doublestar and binary tree to generate the Basic-Cipher-text by Inorder traversal: YTRONCU
3. Make the Cipher-text which is calculated by adding the vertex labels of the modified graceful
labelling to the letters of the Plain-text in reverse order (using the Basic-Cipher-text) as follows:
$\mathrm{Y}+2=\mathrm{A}, \mathrm{T}+3=\mathrm{W}, \mathrm{R}+4=\mathrm{V}, \mathrm{O}+1=\mathrm{P}, \mathrm{N}+5=\mathrm{T}, \mathrm{C}+$ $0=\mathrm{C}, \mathrm{U}+0=\mathrm{U}$, thus yielding the Cipher-text: A W V P T C U (only the Cipher-text needs to be sent to the receiver; calculations of the Cipher-text is only for demonstration purpose)
4. Make the Cipher-Matrix-Doublestar as follows:
Cipher-Matrix-Doublestar

| Y | 2 | x 1 | $\mathrm{~A}=\mathrm{Y}+2$ |
| :--- | :--- | :--- | :--- |
| T | 3 | x 3 | $\mathrm{~W}=\mathrm{T}+3$ |
| R | 4 | x 2 | $\mathrm{~V}=\mathrm{R}+4$ |
| O | 1 | x 6 | $\mathrm{P}=\mathrm{O}+1$ |
| N | 5 | x 4 | $\mathrm{~T}=\mathrm{N}+5$ |
| C | 0 | x 7 | $\mathrm{C}=\mathrm{C}+0$ |
| U | 0 | x 5 | $\mathrm{U}=\mathrm{U}+0$ |

5. Send the Cipher-text, Cipher-Matrix-Doublestar to the receiver.

## Decryption (Doublestar):

1. Arrange xi's in ascending order of subscripts: $\mathrm{x} 1, \mathrm{x} 2, \mathrm{x} 3$, x4, x5, x6, x7 to get AVWTUPC
2. Arrange the above in reverse order to get CPUTWVA
3. Perform subtraction to obtain the original Plain-text as follows:
$\mathrm{C}-0=\mathrm{C}, \mathrm{P}-1=\mathrm{O}, \mathrm{U}-0=\mathrm{U}, \mathrm{T}-5=\mathrm{N}, \mathrm{W}-3=\mathrm{T}, \mathrm{V}-4$ $=\mathrm{R}, \mathrm{A}-2=\mathrm{Y}$ therefore yielding the original Plain-text: COUNTRY

## C. Kruskal's Algorithm

The definitions of the concepts used are as follows:

Weighted Graph: A weight is a non-negative integer assigned to an edge of a graph which could mean an entity, for instance - the distance between the two end-vertices of that edge. Such a graph with weights assigned to its edges is called a weighted graph.

Spanning Tree: A spanning tree of a graph $G$ is its subgraph which contains all the vertices of G and is a tree (connected and acyclic) in itself.

Minimal Spanning Tree: The minimally weighted spanning tree of a weighted graph is called a minimal spanning tree. Kruskal's algorithm gives a method to find minimal spanning tree of a graph.
Rooted Tree: A rooted tree has got a fixed vertex called root from which all other edges go downwards.

Modified Ordered Rooted Tree: An ordered rooted tree numbers edges in a rooted tree from left to right in ascending order at each level of the rooted tree. In this work, in the newly defined modified rooted tree, the pre-assigned edge labels are read from left to right at each level of a rooted tree starting from top level to bottom level and the labels are not necessarily in ascending order.

The letters of the Plain-text: CRYPTOLOGICS are assigned to the edges labelled with $1,2,3,4,5,6,7,8,9,10$, 11,12 with C, R, Y, P, T, O, L, O, G, I, C, S respectively in


Figure 6: Original Graph


Figure 7: Minimal Spanning Tree of the Original Graph by Kruskal's Algorithm

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Figure 8: Modified Ordered Rooted Tree of the Minimal Spanning Tree

Newly developed Cipher-Matrix of the remaining graph (original graph without the Minimal spanning Tree) is shown below where the matrix entry comprises of the pair (edge label, letter of Plain-text).
A, B, C, D, E, F, G, H are the vertex labels of the Original Graph. For example, in this matrix, the matrix entry ( $8, \mathrm{O}$ ) appears as the entry for the edge B-C with end-vertices named with B and C as per the Original Graph.

|  | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $[$ | - | - | - | - | - | - | - |
| B | - | - | $(8,0)$ | $(7, L)$ | - | - | - | - |
| C | - | $(8,0)$ | - | - | - | $(3, Y)$ | - | - |
| D | - | (7,L) | - | - | - | - | - | - |
| E | - | - | - | - | - | - | - | - |
| F | - | - | (3,Y) | - | - | - ( | $(4, P)$ | - |
| G | - | - | - | - | - | (4,P) - | - | $(5, T)$ |
| H | - | - | - | - | - | - (5, | 5,T) | - |

Cipher-Matrix of the remaining graph (original graph without the Minimal spanning Tree)

## Encryption:

1. Associate the numbers in the order of appearance of Plain-text as follows to make the Cipher-
text:

$$
\begin{aligned}
& \mathrm{C}+1=\mathrm{D} \\
& \mathrm{R}+2=\mathrm{T} \\
& \mathrm{Y}+3=\mathrm{B} \\
& \mathrm{P}+4=\mathrm{T} \\
& \mathrm{~T}+5=\mathrm{Y} \\
& \mathrm{O}+6=\mathrm{U} \\
& \mathrm{~L}+7=\mathrm{S} \\
& \mathrm{O}+8=\mathrm{W} \\
& \mathrm{G}+9=\mathrm{P} \\
& \mathrm{I}+10=\mathrm{S} \\
& \mathrm{C}+11=\mathrm{N} \\
& \mathrm{~S}+12=\mathrm{E}
\end{aligned}
$$

Therefore, The Cipher-text is: DTBTYUSWPSNE
2. Make the Minimal Spanning Tree
3. Make a Modified Ordered Rooted Tree of the Minimal Spanning Tree (weights/numbers/ edge labels are
read from left to right at each level from top level to bottom level)
4. Encryption- Array: (N,11), (S,10), (E,12), (P,9), (U,6), (T,2), (D,1) in the order of left to right in the Modified Ordered Rooted Tree. For example, the pair $(\mathrm{N}, 11)$ has been constructed from $\mathrm{C}+11=\mathrm{N}$.
5. Send the Cipher-text, Encryption-Array andCipherMatrix of the remaining graph (original graph without the Minimal Spanning Tree) to the receiver.

## Decryption:

1. From the remaining graph (B,3), (T,4), (Y,5),
(S,7), (W,8) [ pairs constructed as in step 4. of
Encryption for the remaining graph from $\mathrm{Y}+3=\mathrm{B}, \mathrm{P}+4$ $=\mathrm{T}, \mathrm{T}+5=\mathrm{Y}, \mathrm{L}+7=\mathrm{S}$ and
$\mathrm{O}+8=\mathrm{W}$ respectively.], construct letters of the Plain-text: $\mathrm{Y}, \mathrm{P}, \mathrm{T}, \mathrm{L}, \mathrm{O}$ (this can be seen in
the Cipher-Matrix form of the remaining graph also). For example, from $\mathrm{Y}+3=\mathrm{B}$ we get the character Y of the original Plain-text as $\mathrm{Y}=\mathrm{B}-3$ and similarly, $\mathrm{P}=\mathrm{T}-4, \mathrm{~T}=$ $\mathrm{Y}-5, \mathrm{~L}=\mathrm{S}-7, \mathrm{O}=\mathrm{W}-8$ from the Cipher-Matrix i.e., the letters Y, P, T, L, O.
2. Similarly from Encryption- Array also, construct letters of Plain-text: C, I, S, G, O, R, C. For
example, $\mathrm{C}=\mathrm{N}-11, \mathrm{I}=\mathrm{S}-10, \mathrm{~S}=\mathrm{E}-12$ from $\mathrm{C}+11=$ $\mathrm{N}, \mathrm{I}+10+\mathrm{S}, \mathrm{S}+12=\mathrm{E}$ respectively.
3. Regarding the $2^{\text {nd }}$ element of each ordered pair from Encryption- Array and remaining graph, arrange the $2^{\text {nd }}$ elements [ from step 4. of Encryption and step 1. of Decryption taken together]:- the numbers in ascending order and the corresponding letters of Plain-text also will be simultaneously arranged in ascending order giving the original Plain-text.
(D,1), (T,2), (B,3), (T,4), (Y,5), (U,6), (S,7), (W,8), (P,9), (S,10), (N,11), (E,12)

The Plain-text CRYPTOLOGICS is obtained as follows:
D-1 $=\mathrm{C}$
$\mathrm{T}-2=\mathrm{R}$
B-3 $=\mathrm{Y}$
$\mathrm{T}-4=\mathrm{P}$
$\mathrm{Y}-5=\mathrm{T}$
$\mathrm{U}-6=\mathrm{O}$
$\mathrm{S}-7=\mathrm{L}$
$\mathrm{W}-8=0$
P-9 = G
S $-10=\mathrm{I}$
$\mathrm{N}-11=\mathrm{C}$
$\mathrm{E}-12=\mathrm{S}$
$A N Y$ numbers can be assigned to the letters of the Plaintext such as $5,10,16 \ldots$ and we can proceed similarly with an infinite number of Cipher-texts.


A variety of operations such as addition, subtraction, addition modulo 7 ( 7 is the number of vertices) or any other combination of multiplication modulo 7 and addition modulo 7 can be taken to hide the original message still further and in decryption, the reverse operation will be done thus yielding an infinite number of Cipher-texts impossible or too difficult to crack. The approach of modified operations like the modulo arithmetic can be used in the other methods discussed previously in the subsections A and $B$ also to hide the Plain-text even more in the encryption, and in decryption, the reverse operation can be done. For demonstration purpose, addition and subtraction have been shown.

## III. CONCLUSION

In this work, the tree traversal algorithms like Inorder, Preorder, Postorder yielding the Infix, Prefix and Postfix notations respectively have been demonstrated for resulting in Cipher-texts with and without expression trees. Kruskal's algorithm for making minimal spanning tree, constructing the conceptually new modified ordered rooted tree of the minimal spanning tree and making use of the newly developed Cipher-Matrix of the remaining graph (original graph without the minimal spanning tree) have been presented. Also, the modified graceful labelling allowing repetition of exactly one vertex label for certain graphs like the binary tree and doublestar, has been employed to create highly hidden Cipher-texts. The idea of Basic-Cipher-text also has been used to hide the Plain-text even more and has been demonstrated in the tree traversal algorithms and for the doublestar graph. All these concepts and methods yield highly cryptic Cipher-texts impossible or too difficult to crack. Encryption and decryption algorithms for all these concepts have been presented in this work for their practical Cryptology applications in securing an enhanced safety in data transfer.

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